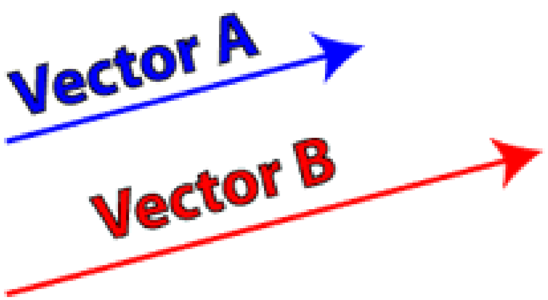
Vectors and Scalars



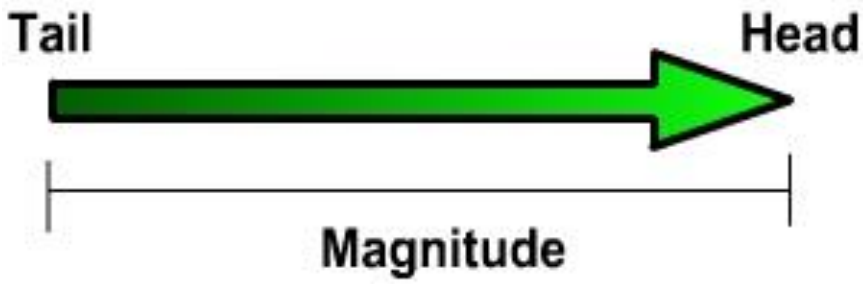
Scalar

- A **SCALAR** is ANY quantity in physics that has MAGNITUDE, but NOT a direction associated with it.
- Magnitude – A numerical value with units.

Scalar Example	**Magnitude**
Speed	20 m/s
Distance	10 m
Age	15 years
Heat	1000 calories

Describing Vectors

- Vectors are usually represented by a line
- with an arrow on one end.
- The end without the arrow is called the
- TAIL.
- The end with the arrow on it is called the
- HEAD.

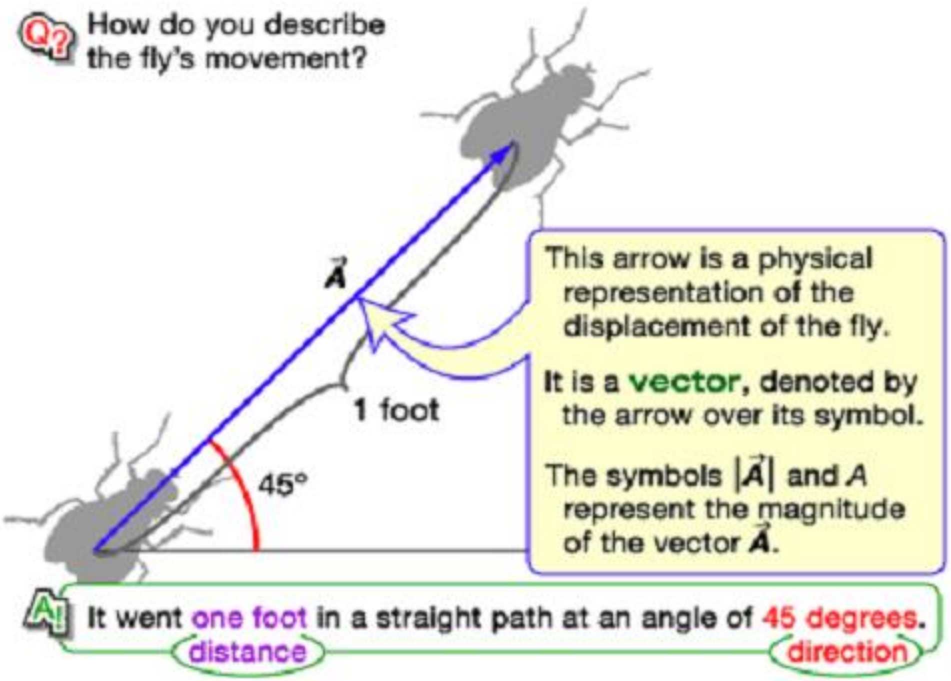


Vector

A **VECTOR** is ANY quantity in physics that has BOTH MAGNITUDE and DIRECTION.

Vector	Magnitude & Direction
Velocity	20 m/s, N
Acceleration	10 m/s/s, E
Force	5 N, West

$$\vec{v}, \vec{x}, \vec{a}, \vec{F}$$



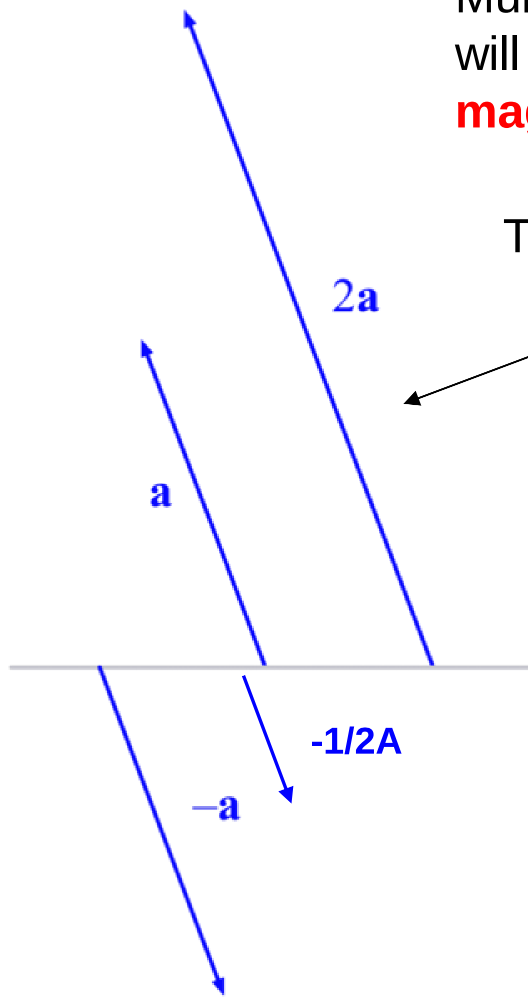
Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.

Scalar Multiplication

Multiplying a vector by a scalar will ONLY CHANGE its **magnitude**.

Thus if **A = 12**, Then **2A = 24**

Multiplying a vector by "-1" does not change the magnitude, but it does reverse it's direction or in a sense, it's angle.

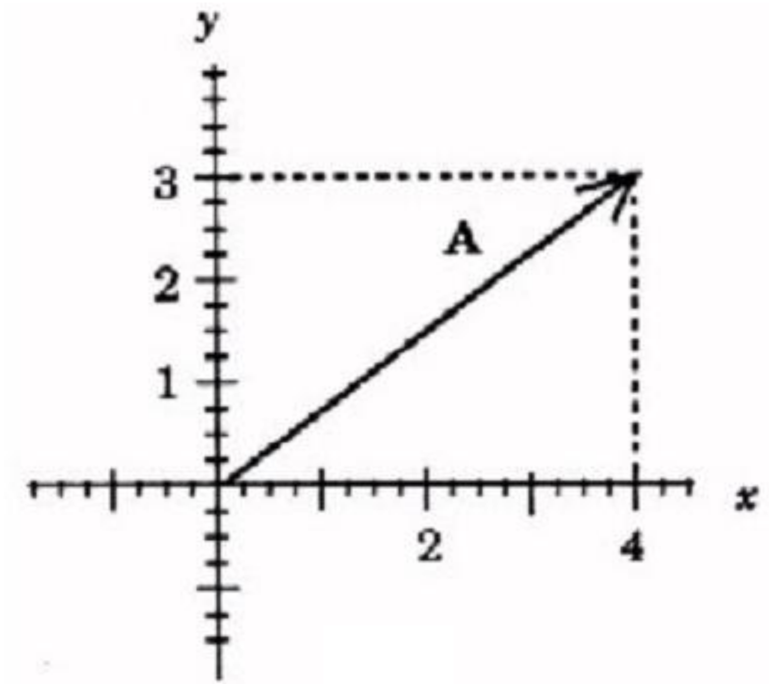


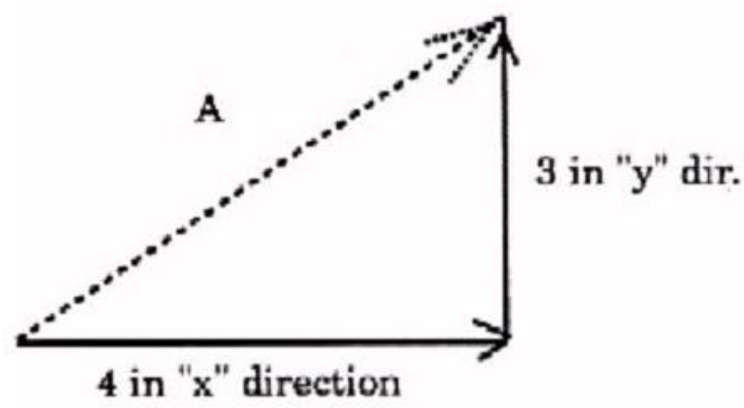
Thus if **A = 12**, then **-A = -12**

If **A = 12**, then **(-1/2)A = -6**

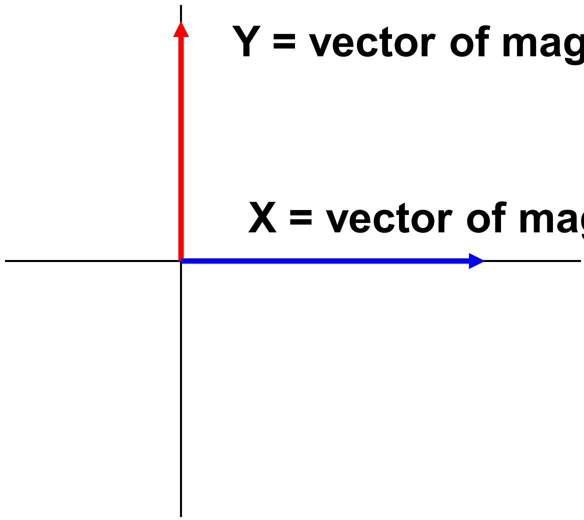
Unit Vector Notation

An effective and popular system used in engineering is called **unit vector notation**. It is used to denote vectors with an x-y Cartesian coordinate system.



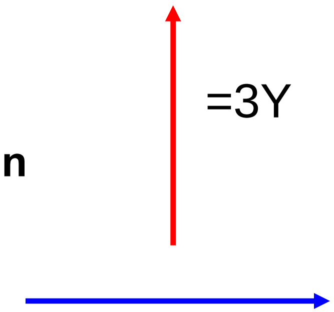


Unit Vector Notation



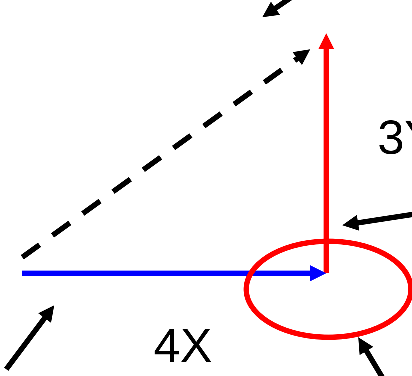
Y = vector of magnitude “1” in the “y” direction

X = vector of magnitude “1” in the “x” direction



The hypotenuse in Physics is called the **RESULTANT or VECTOR SUM.**

The LEGS of the triangle are called the **COMPONENTS**



Vertical Component

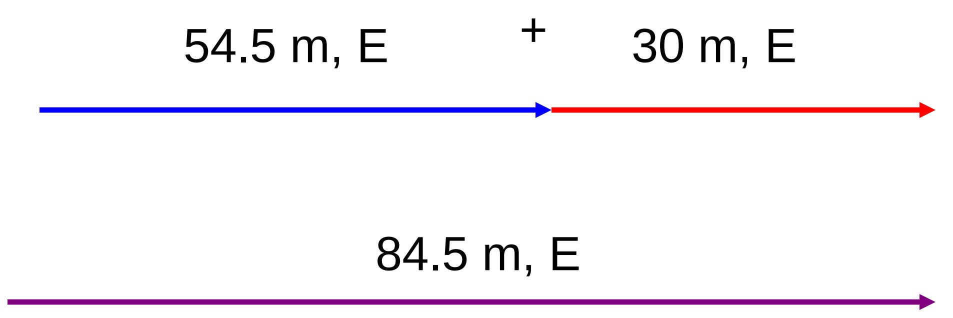
Horizontal Component

NOTE: When drawing a right triangle that conveys some type of motion, <u>you MUST draw your components **HEAD TO TOE**.</u>

Applications of Vectors

VECTOR ADDITION – If 2 similar vectors point in the SAME direction, add them.

- **Example: A man walks 54.5 meters east, then another 30 meters east. Calculate his displacement relative to where he started?**

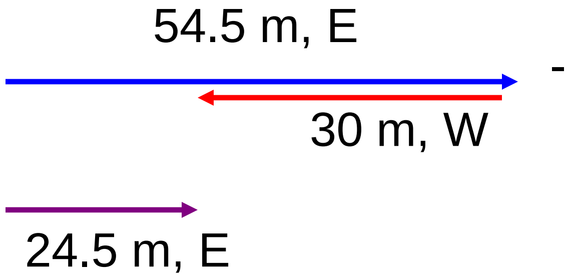


Notice that the SIZE of the arrow conveys MAGNITUDE and the way it was drawn conveys DIRECTION.

Applications of Vectors

VECTOR SUBTRACTION - If 2 vectors are going in opposite directions, you **SUBTRACT**.

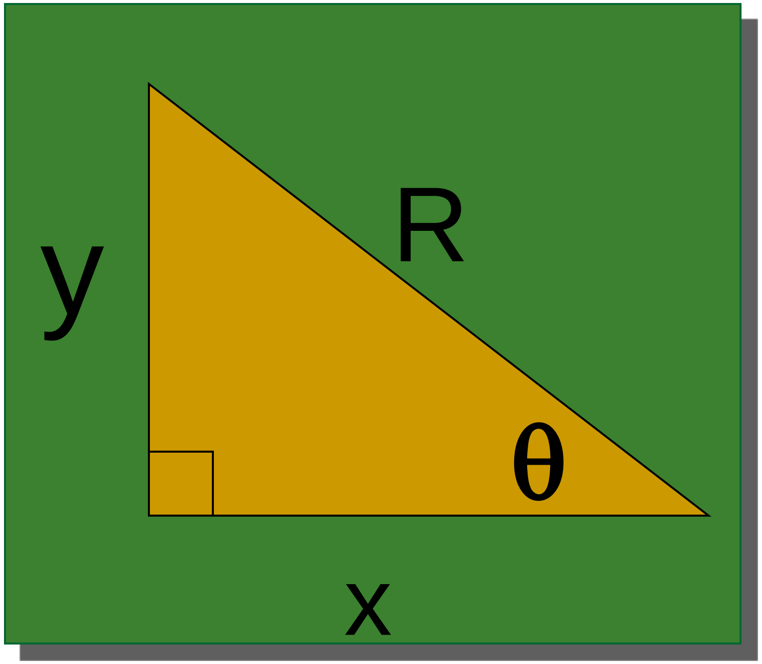
- **Example: A man walks 54.5 meters east, then 30 meters west. Calculate his displacement relative to where he started?**



Ever hear of SOH-CAH-TOA?

- To get direction, we need to find out
- what angle she is moving at.
- We can also use trigonometric
- equations to find the other angles of
- the triangle.
- Any of the following may be used.
- **S**in θ = **O**pposite/**H**ypotenuse
- **C**os θ = **A**djacent / **H**ypotenuse
- **T**an θ = **O**pposite / **A**djacent
- all angles of a triangle = 180°

- You must have mastered right-triangle trigonometry.



Sin θ = Y/R

$\boxed{Y = R \sin \theta}$

Cos θ = X/R

$\boxed{X = R \cos \theta}$

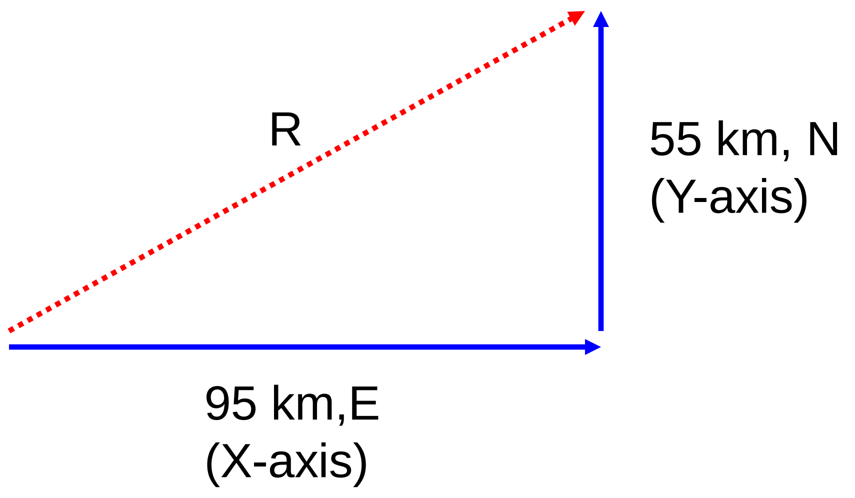
Tan θ = Y/X

$\boxed{R^2 = x^2 + y^2}$

Non-Collinear Vectors

When 2 vectors are perpendicular, you must use the Pythagorean theorem.

A man walks 95 km, East then 55 km, north. Calculate his RESULTANT DISPLACEMENT.



$$r^2 = x^2 + y^2 \rightarrow r = \sqrt{x^2 + y^2}$$

$$r = \text{Resultant} = \sqrt{95^2 + 55^2}$$

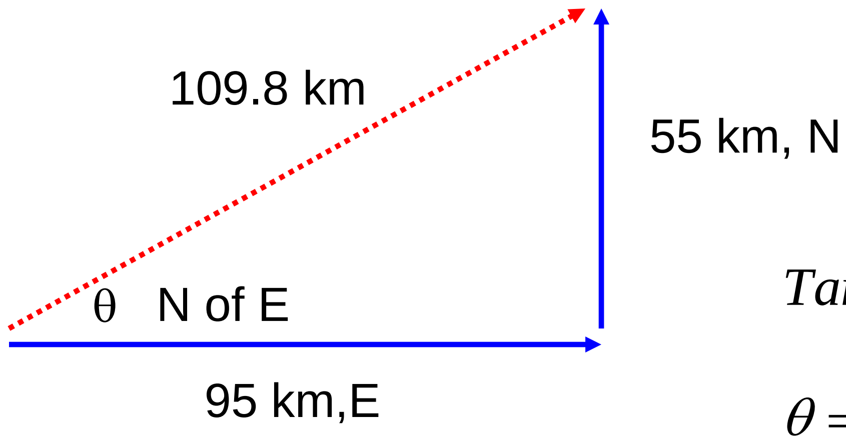
$$r = \sqrt{12050} = 109.8\ km$$

Heads up.

- If you are using a Ti calculator press Mode and select Degree. We will be calculating angles now.

BUT…..what about the VALUE of the angle???

Just putting North of East on the answer is NOT specific enough for the direction. We MUST find the VALUE of the angle.



To find the value of the angle we use a Trig function called TANGENT.

$$Tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{55}{95} = 0.5789$$

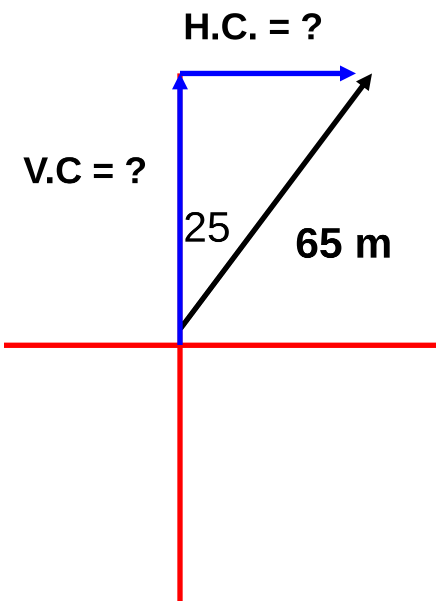
$$\theta = Tan^{-1}(0.5789) = 30^{\circ}$$

So the COMPLETE final answer is :

$$109.8km @ 30^{\circ}\ NofE$$

What if you are missing a component?

Suppose a person walked 65 m, 25 degrees East of North. What were his horizontal and vertical components?



The goal: **ALWAYS MAKE A RIGHT TRIANGLE!**

To solve for components, we often use the trig functions sine and cosine.

$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

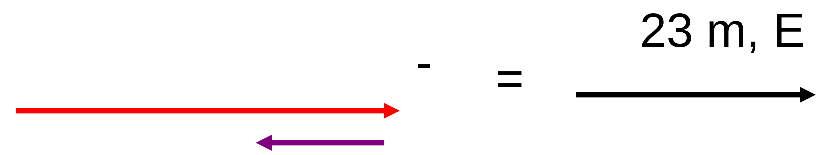
$$adj = hyp * \cos\theta \qquad opp = hyp * \sin\theta$$

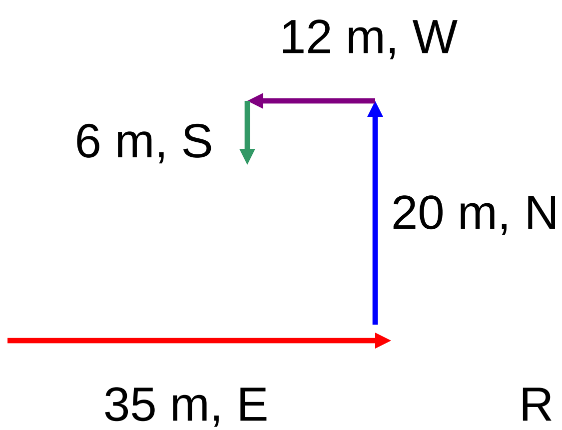
$$adj = V.C. = 65\cos 25 = 58.91m,\ N\ or\ 58.91m$$

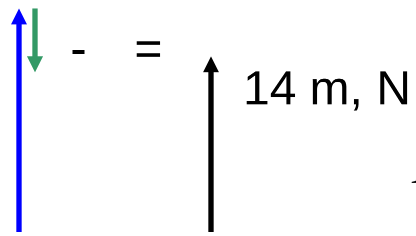
$$opp = H.C. = 65\sin 25 = 27.47m,\ E\ or\ 27.47\,m$$

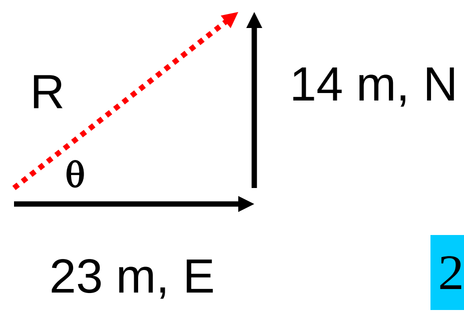
Example

A bear, searching for food wanders 35 meters east then 20 meters north. Frustrated, he wanders another 12 meters west then 6 meters south. Calculate the bear's displacement.









$$R = \sqrt{14^2 + 23^2} = 26.93m$$

$$Tan\theta = \frac{14}{23} = .6087$$

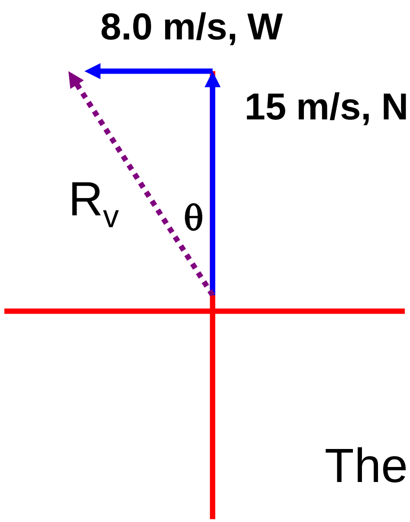
$$\theta = Tan^{-1}(0.6087) = 31.3^{\circ}$$

The Final Answer:

$$26.93m @ 31.3^{\circ}\ NofE$$

Example

A boat moves with a velocity of 15 m/s, N in a river which flows with a velocity of 8.0 m/s, west. Calculate the boat's resultant velocity with respect to due north.



$$R_v = \sqrt{8^2 + 15^2} = 17\, m/s$$

$$Tan\theta = \frac{8}{15} = 0.5333$$

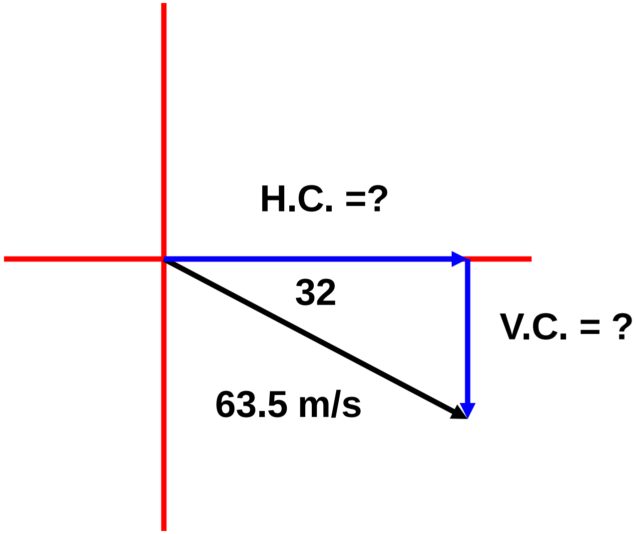
$$\theta = Tan^{-1}(0.5333) = 28.1^\circ$$

The Final Answer :

$$17\, m/s @ 28.1^\circ\, WofN$$

Example

A plane moves with a velocity of 63.5 m/s at 32 degrees South of East. Calculate the plane's horizontal and vertical velocity components.



$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = H.C. = 63.5\cos 32 = 53.85\,m/s, E\ or\ +53.85m$$

$$opp = V.C. = 63.5\sin 32 = 33.64\,m/s, S\ or\ -33.64m$$