SIGNIFICANT DIGITS

The number of significant digits in an answer to a calculation will depend on the number of significant digits in the given data, as discussed in the rules below. *Approximate* calculations (order-of-magnitude estimates) always result in answers with only one or two significant digits.

When are Digits Significant?

Non-zero digits are always significant. Thus, 22 has two significant digits, and 22.3 has three significant digits.

With zeroes, the situation is more complicated:

a. Zeroes placed before other digits are not significant; 0.046 has two significant digits.
b. Zeroes placed between other digits are always significant; 4009 kg has four significant digits.
c. Zeroes placed after other digits but behind a decimal point are significant; 7.90 has three significant digits.
d. Zeroes at the end of a number are significant only if they are behind a decimal point as in (c). Otherwise, it is impossible to tell if they are significant. For example, in the number 8200, it is not clear if the zeroes are significant or not. The number of significant digits in 8200 is at least two, but could be three or four. To avoid uncertainty, use scientific notation to place significant zeroes behind a decimal point:

$$8.200 \times 10^3 \text{ has four significant digits}$$

$$8.20 \times 10^3 \text{ has three significant digits}$$

$$8.2 \times 10^3 \text{ has two significant digits}$$

Significant Digits in Multiplication, Division, Trig. functions, etc.

In a calculation involving multiplication, division, trigonometric functions, etc., the number of significant digits in an answer should equal the least number of significant digits in any one of the numbers being multiplied, divided etc.

Thus in evaluating $\sin(kx)$, where $k = 0.097 \text{ m}^{-1}$ (two significant digits) and $x = 4.73$ m (three significant digits), the answer should have two significant digits.

Note that whole numbers have essentially an unlimited number of significant digits. As an example, if a hair dryer uses 1.2 kW of power, then 2 identical hairdryers use 2.4 kW:

1.2 kW {2 sig. dig.} $\times$ 2 {unlimited sig. dig.} = 2.4 kW {2 sig. dig.}

Significant Digits in Addition and Subtraction

When quantities are being added or subtracted, the number of *decimal places* (not significant digits) in the answer should be the same as the least number of decimal places in any of the numbers being added or subtracted.

Example:

5.67 J (two decimal places)
1.1 J (one decimal place)
<u>0.9378 J</u> (four decimal place)
7.7 J (one decimal place)

Keep One Extra Digit in Intermediate Answers

When doing multi-step calculations, *keep at least one more significant digit in intermediate results* than needed in your final answer.

For instance, if a final answer requires two significant digits, then carry at least three significant digits in calculations. If you round-off all your intermediate answers to only two digits, you are discarding the information contained in the third digit, and as a result the *second* digit in your final answer might be incorrect. (This phenomenon is known as "round-off error.")

The Two Greatest Sins Regarding Significant Digits

1. Writing more digits in an answer (intermediate or final) than justified by the number of digits in the data.
2. Rounding-off, say, to two digits in an intermediate answer, and then writing three digits in the final answer.