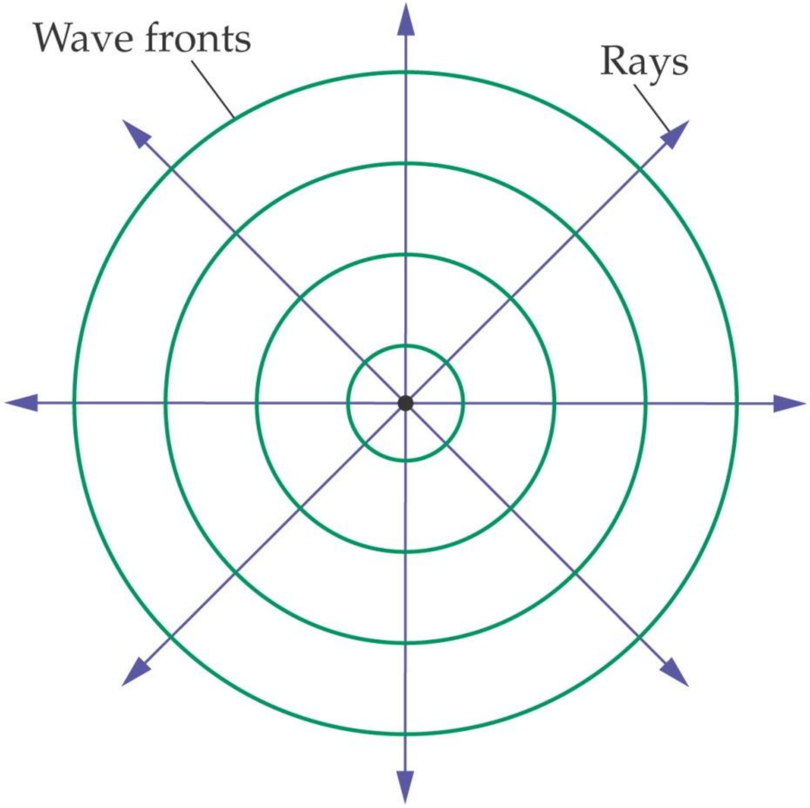
Geometrical Optics

Units

- The Reflection of Light
- Forming Images with a Plane Mirror
- Spherical Mirrors
- Ray Tracing and the Mirror Equation
- The Refraction of Light
- Ray Tracing for Lenses
- The Thin-Lens Equation
- Dispersion and the Rainbow

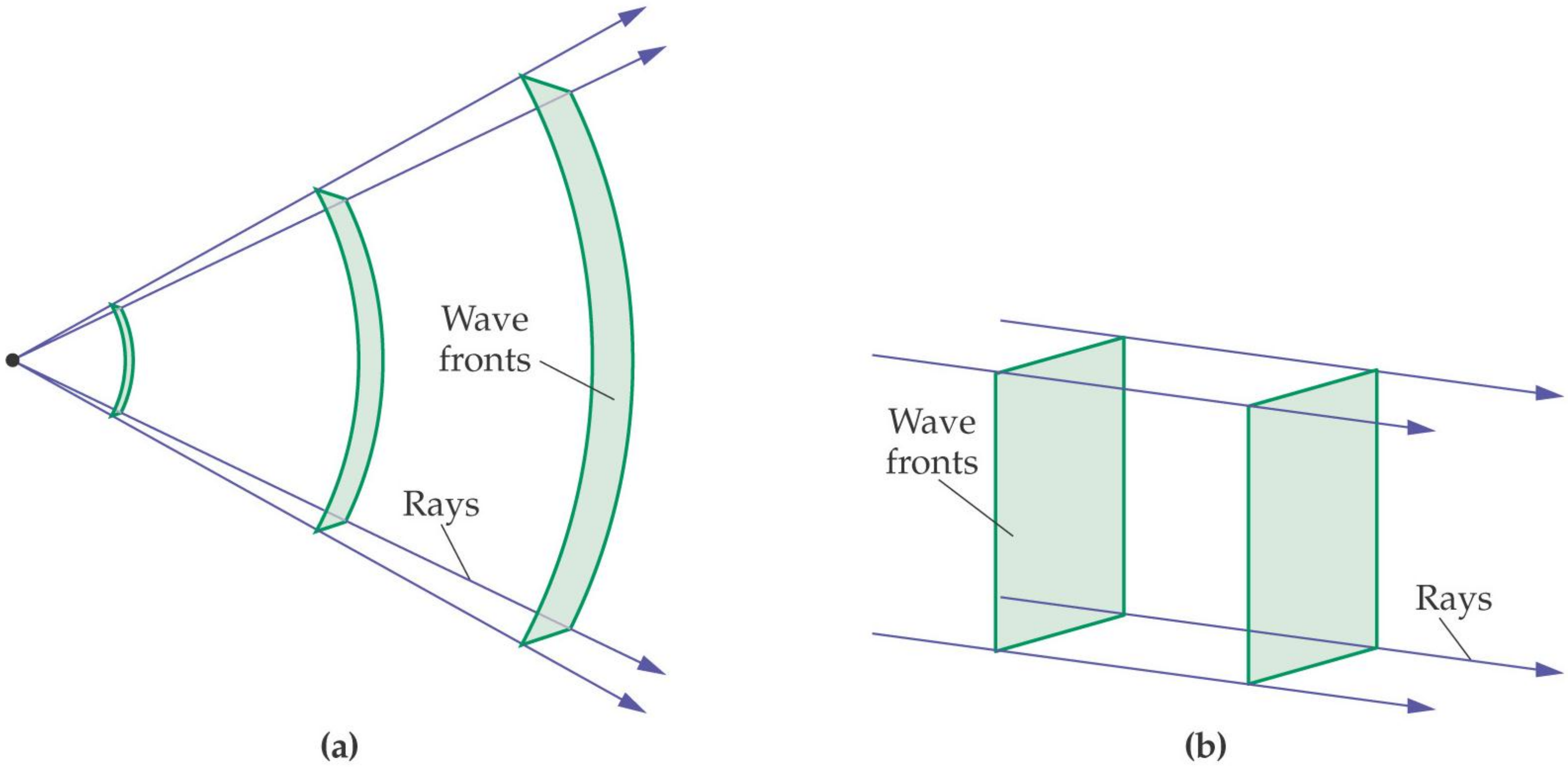
The Reflection of Light

If a stone is dropped into a pond, circular waves emanate from the point where it landed. Rays, perpendicular to the wave fronts, give the direction in which the waves propagate.



The Reflection of Light

As one moves farther from a point wave source, the wave fronts become more nearly flat.

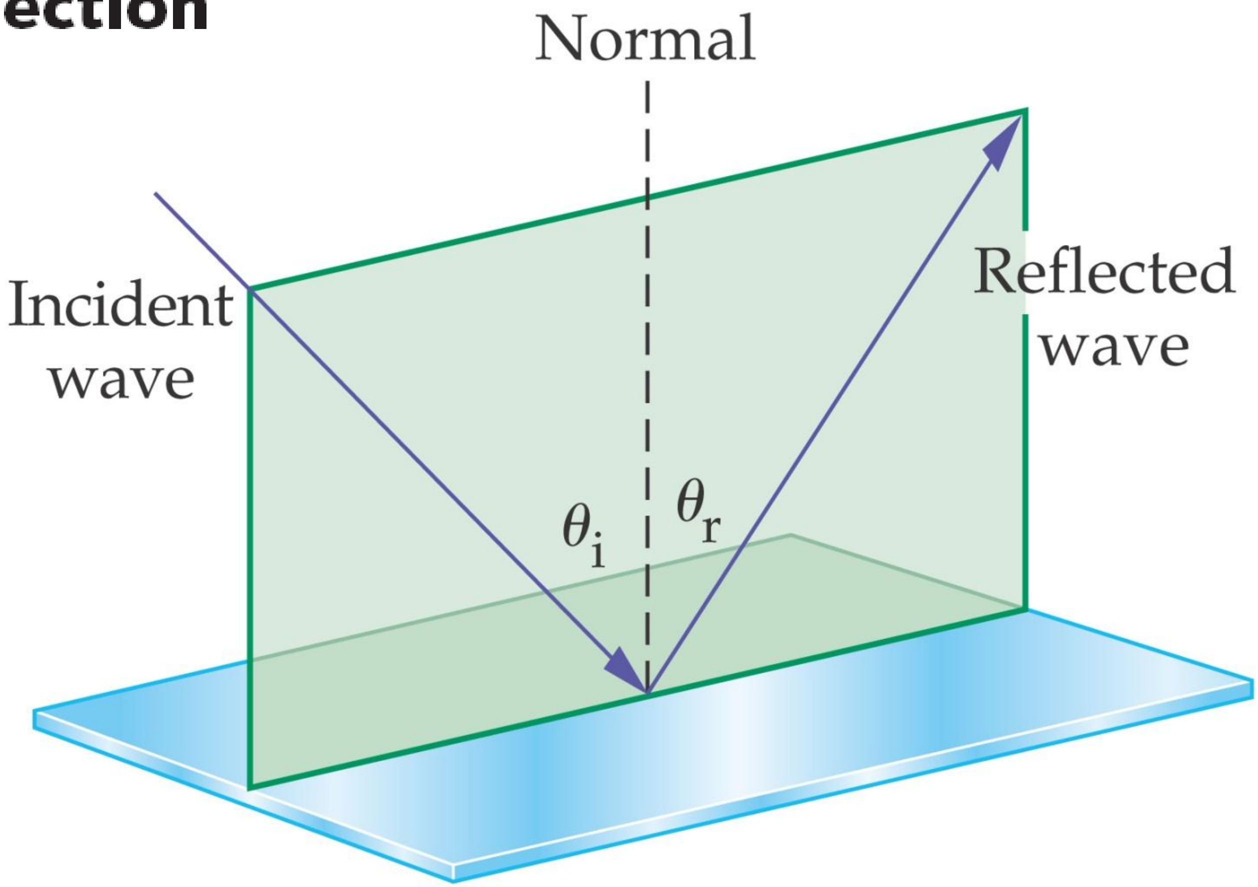


The Reflection of Light

The law of reflection states that the angle of incidence equals the angle of reflection:

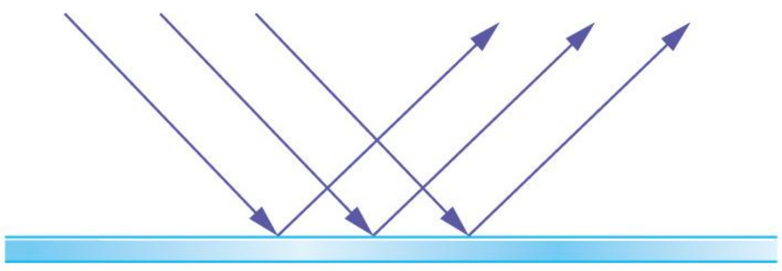
Law of Reflection

$$\theta_r = \theta_i$$

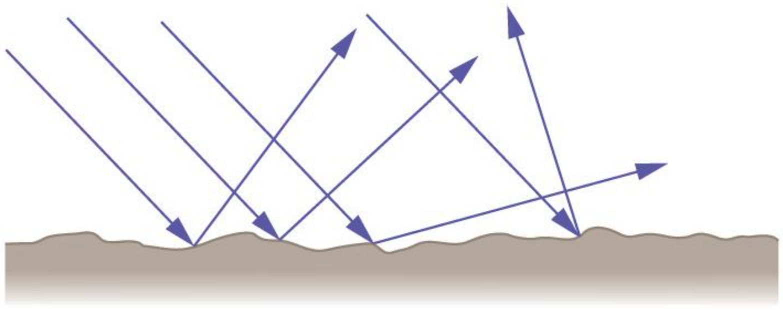


The Reflection of Light

Reflection from a smooth surface is called specular reflection; if the surface is rough, it is diffuse reflection.



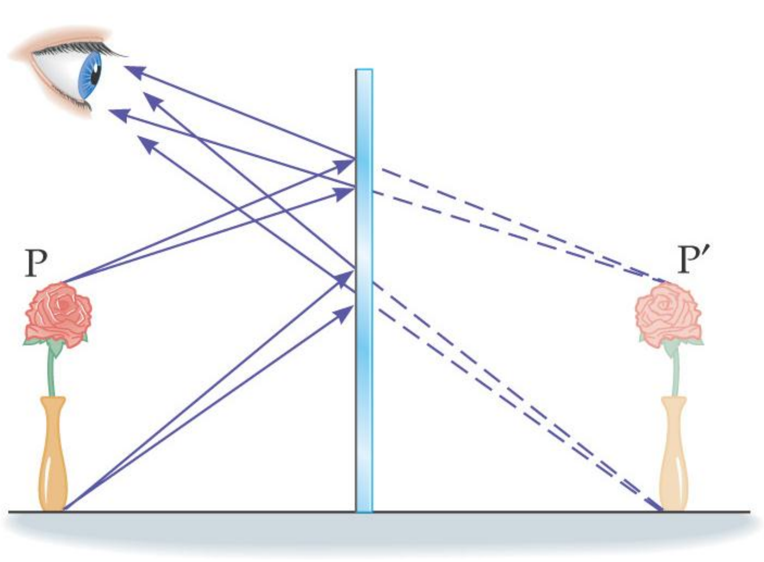
(a) Specular reflection

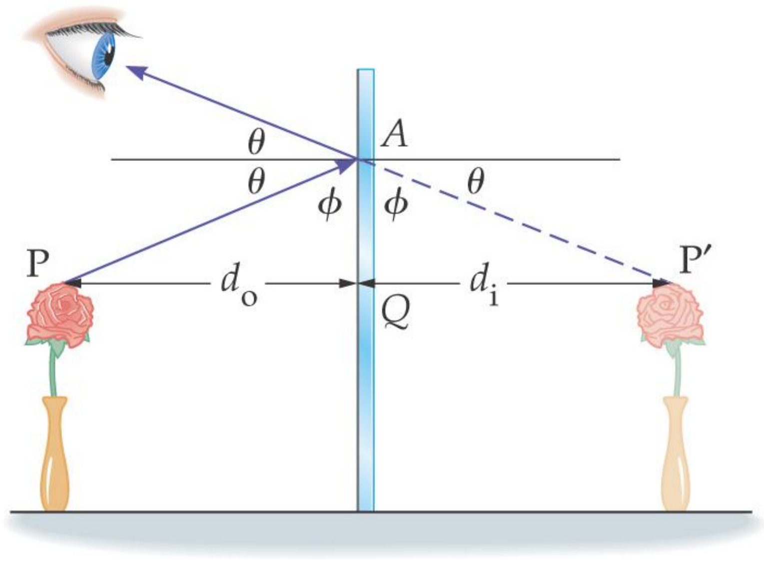


(b) Diffuse reflection

Forming Images with a Plane Mirror

Light reflected from the flower and vase hits the mirror. Obeying the law of reflection, it enters the eye. The eye interprets the ray as having had a straight-line path, and sees the image behind the mirror.





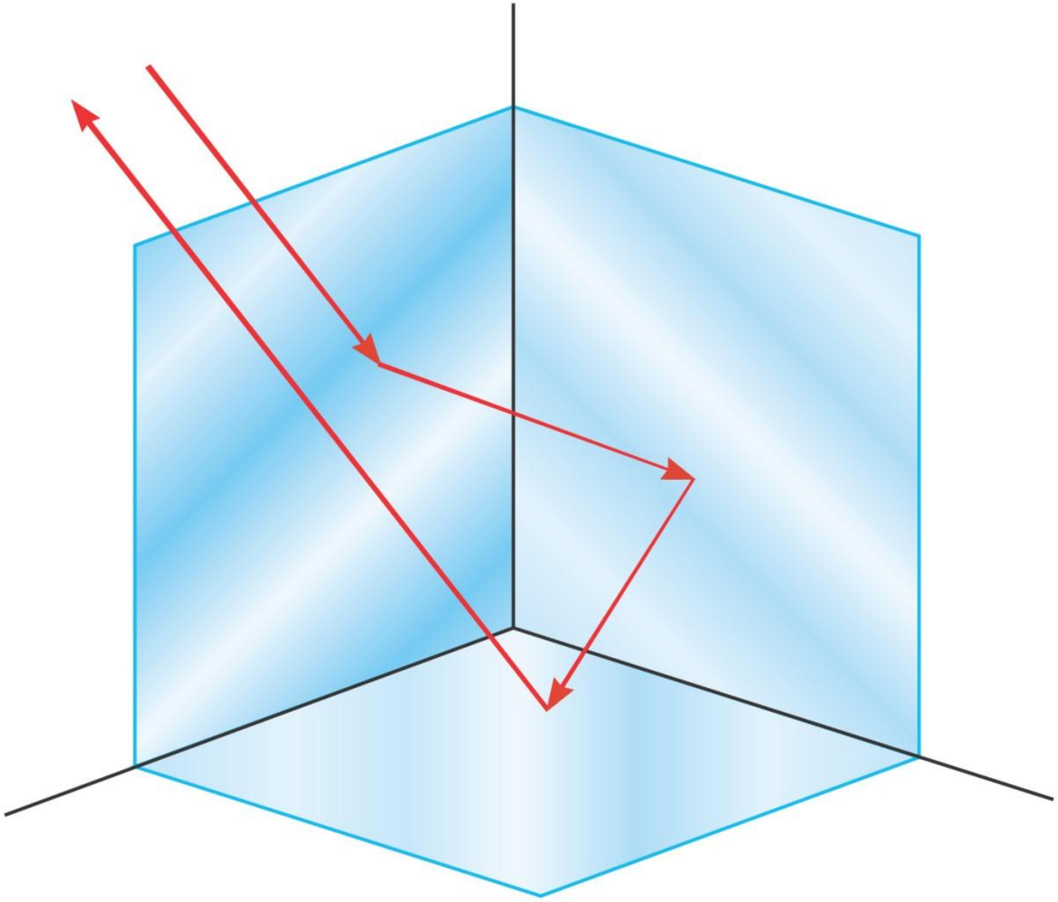
Forming Images with a Plane Mirror

Properties of Mirror Images Produced by Plane Mirrors:

- **A mirror image is upright, but appears reversed right to left.**
- **A mirror image appears to be the same distance behind the mirror that the object is in front of the mirror.**
- **A mirror image is the same size as the object.**

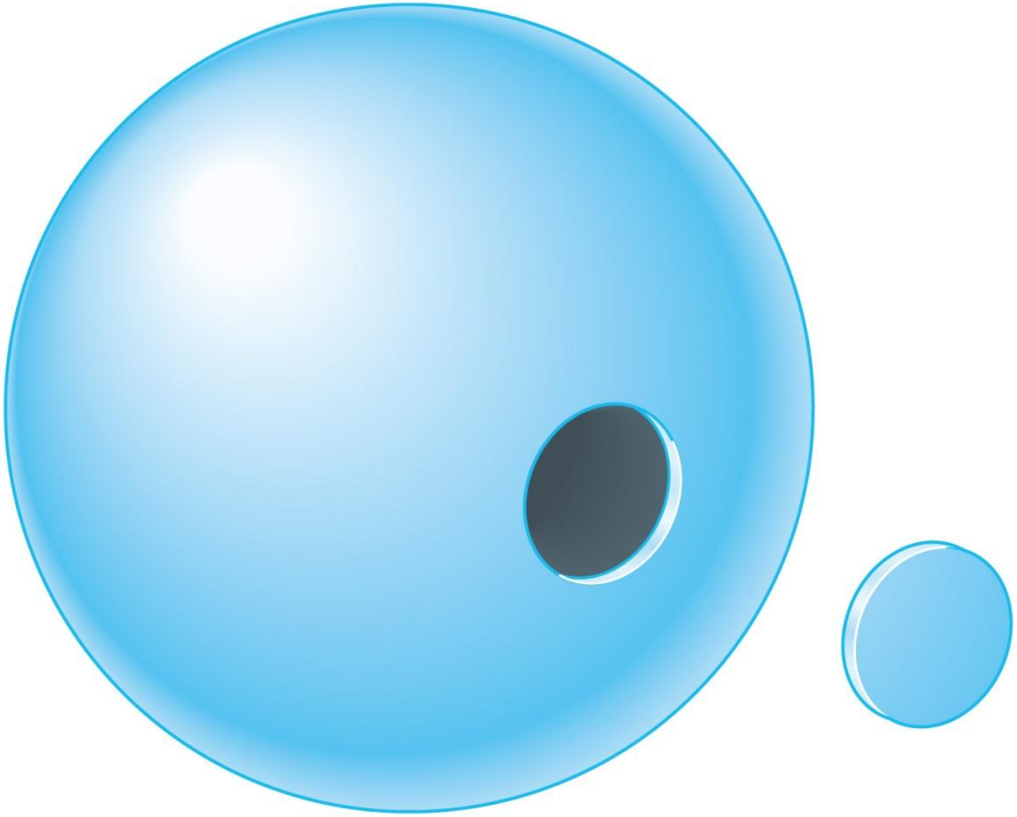
Forming Images with a Plane Mirror

A corner reflector reflects light parallel to the incident ray, no matter the incident angle.



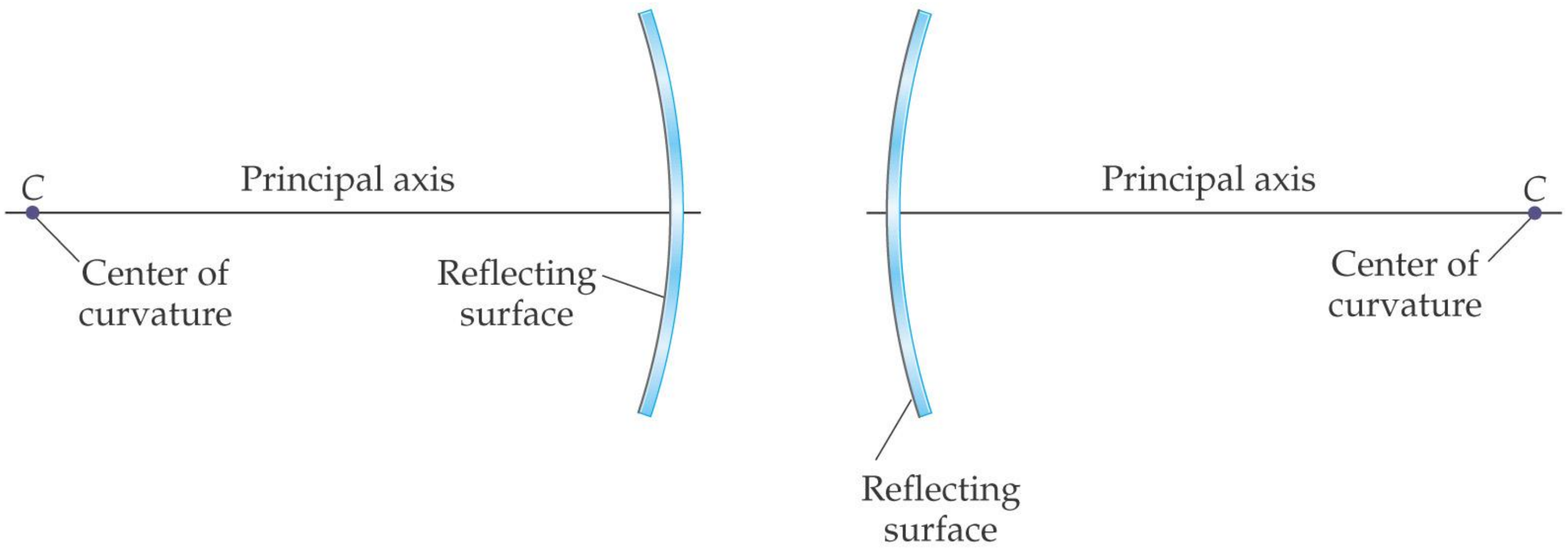
Spherical Mirrors

A spherical mirror has the shape of a section of a sphere. If the outside is mirrored, it is convex; if the inside is mirrored, it is concave.



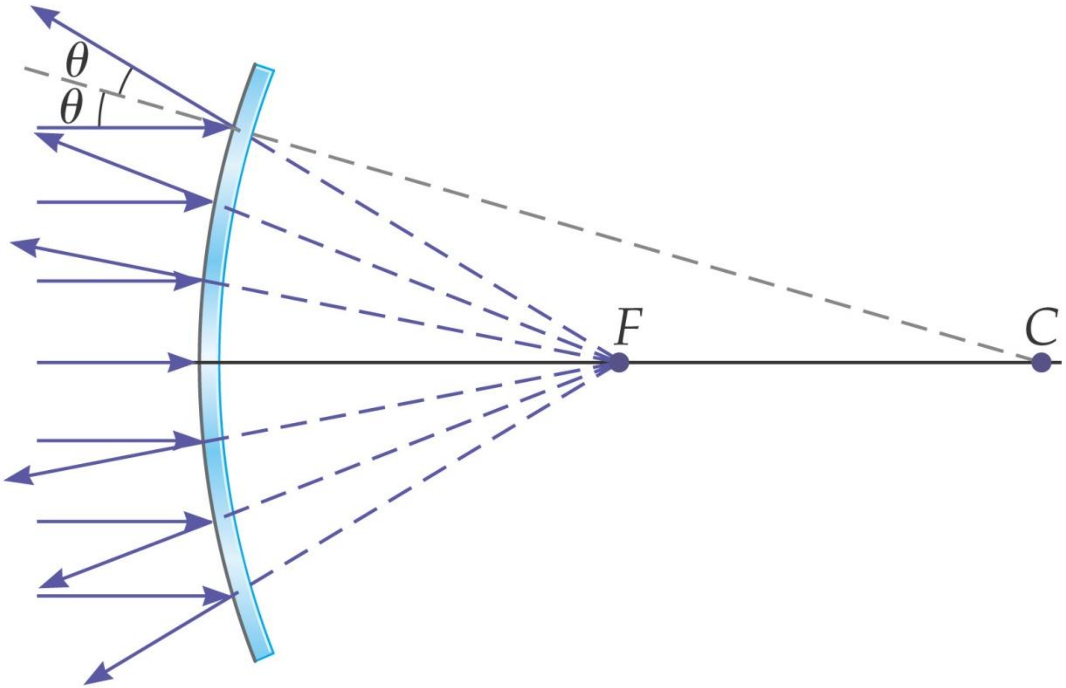
Spherical Mirrors

Spherical mirrors have a central axis (a radius of the sphere) and a center of curvature (the center of the sphere).



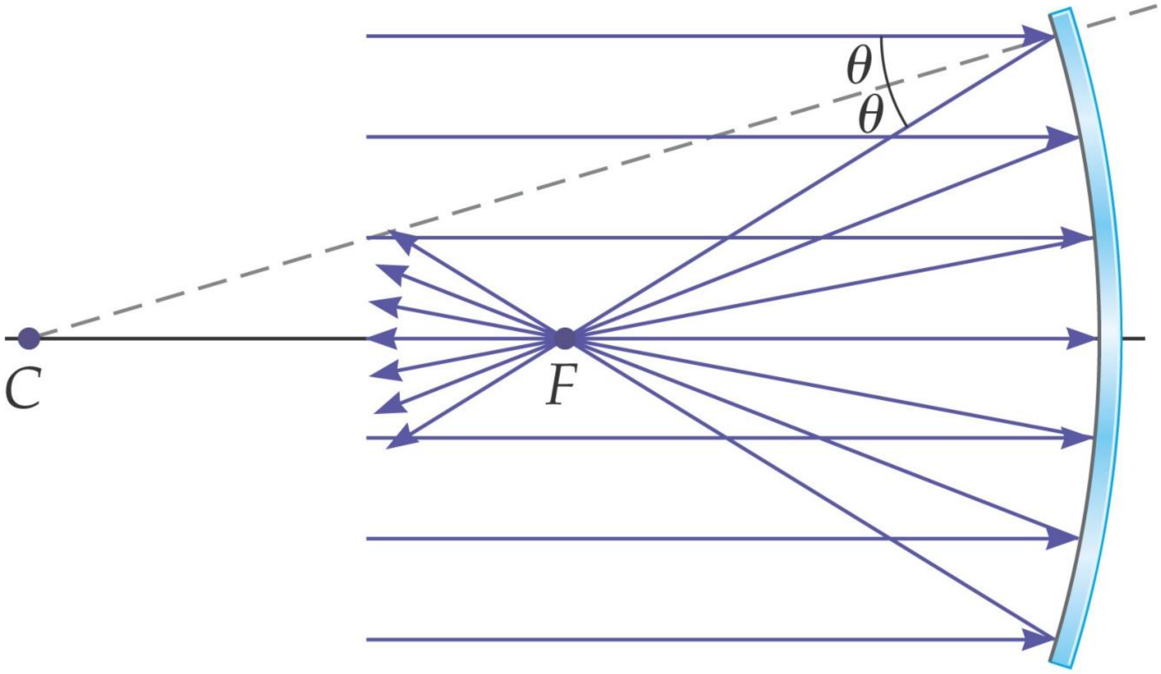
Spherical Mirrors

Parallel rays hitting a spherical mirror come together at the focal point (or appear to have come from the focal point, if the mirror is convex).



Spherical Mirrors

This is a ray diagram for finding the focal point of a concave mirror.



Spherical Mirrors

Focal Length for a Convex *Mirror* of Radius *R*

$$f = -\frac{1}{2}R$$

SI unit: m

For a convex mirror, the focal length is negative, as the rays do not go through the focal point. The opposite is true for a concave mirror.

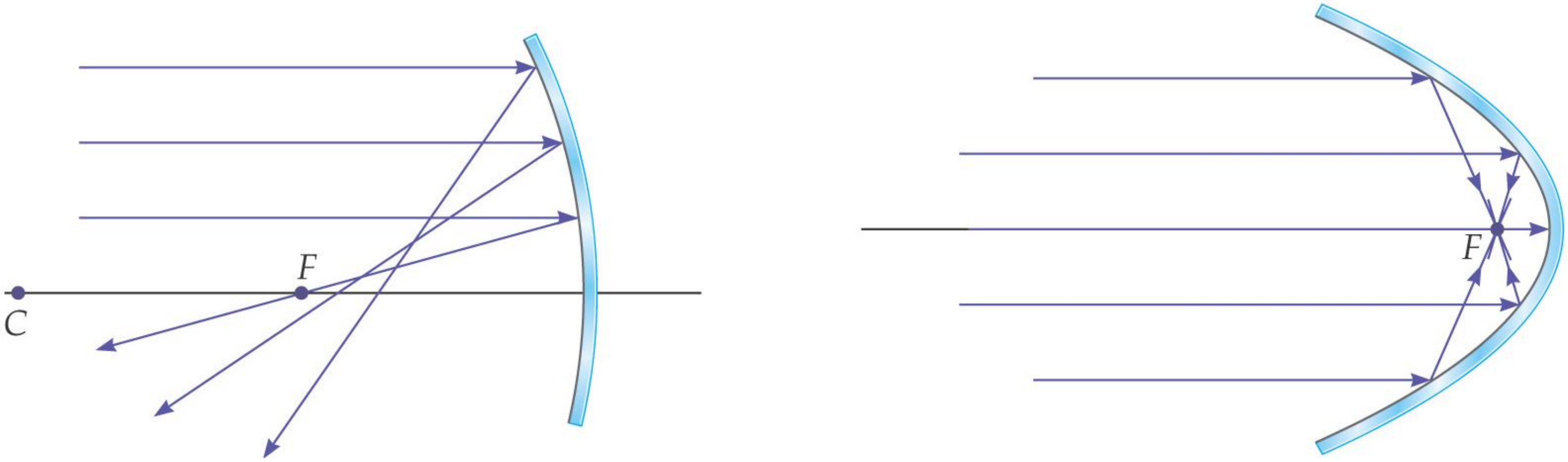
Focal Length for a Concave *Mirror* of Radius *R*

$$f = \frac{1}{2}R$$

SI unit: m

Spherical Mirrors

We have made the assumption here that the rays do not hit the mirror very far from the principal axis. If they do, the image is blurred; this is called spherical aberration, and can be remedied by using a parabolic mirror instead.



Spherical Mirrors

When the Hubble Space Telescope was first launched, its optics were marred by spherical aberration. This was fixed with corrective optics.

Ray Tracing and the Mirror Equation

We use three principal rays in finding the image produced by a concave mirror.

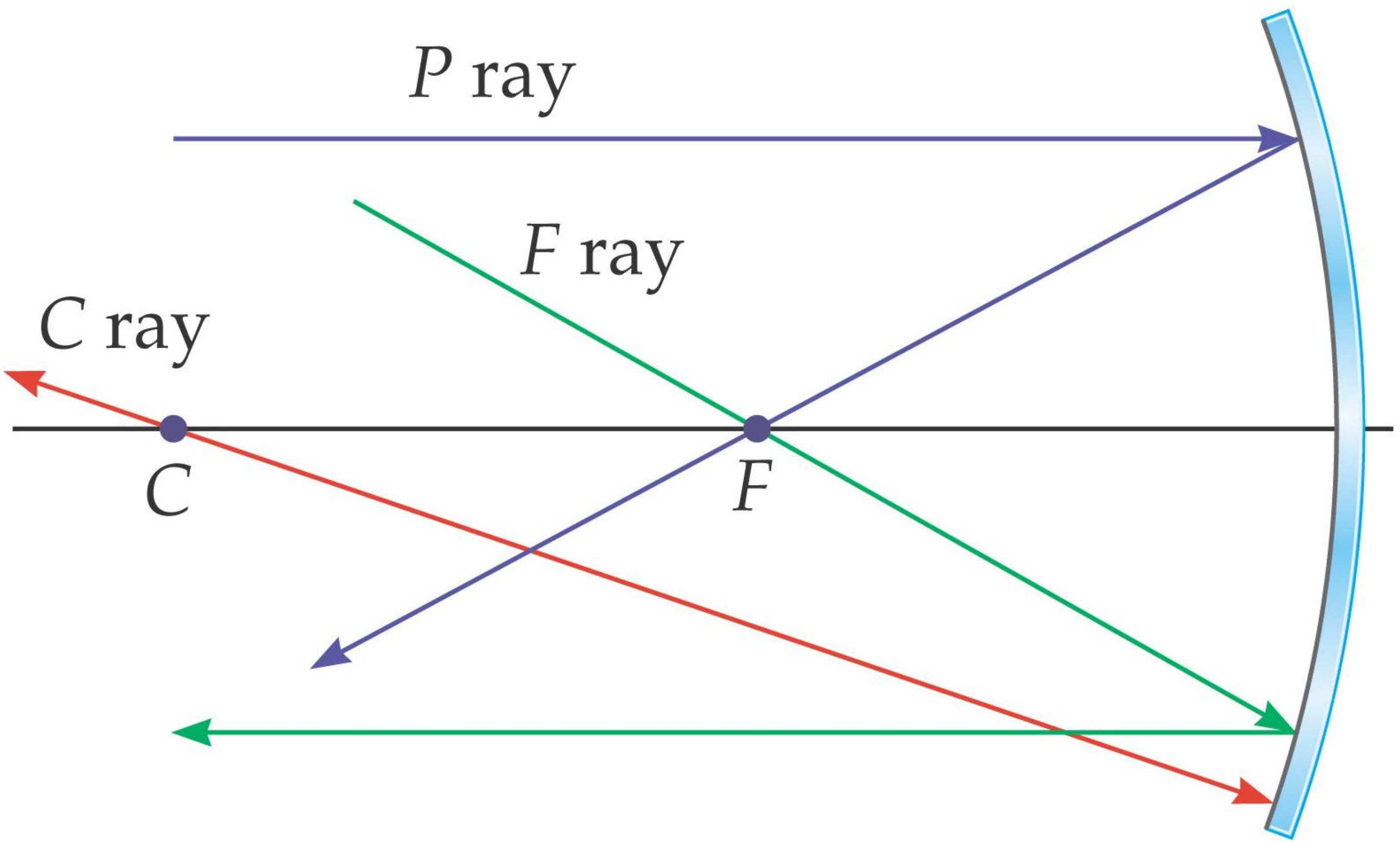
• The parallel ray (P ray) reflects through the focal point.

• The focal ray (F ray) reflects parallel to the axis.

• The center-of-curvature ray (C ray) reflects back along its incoming path.

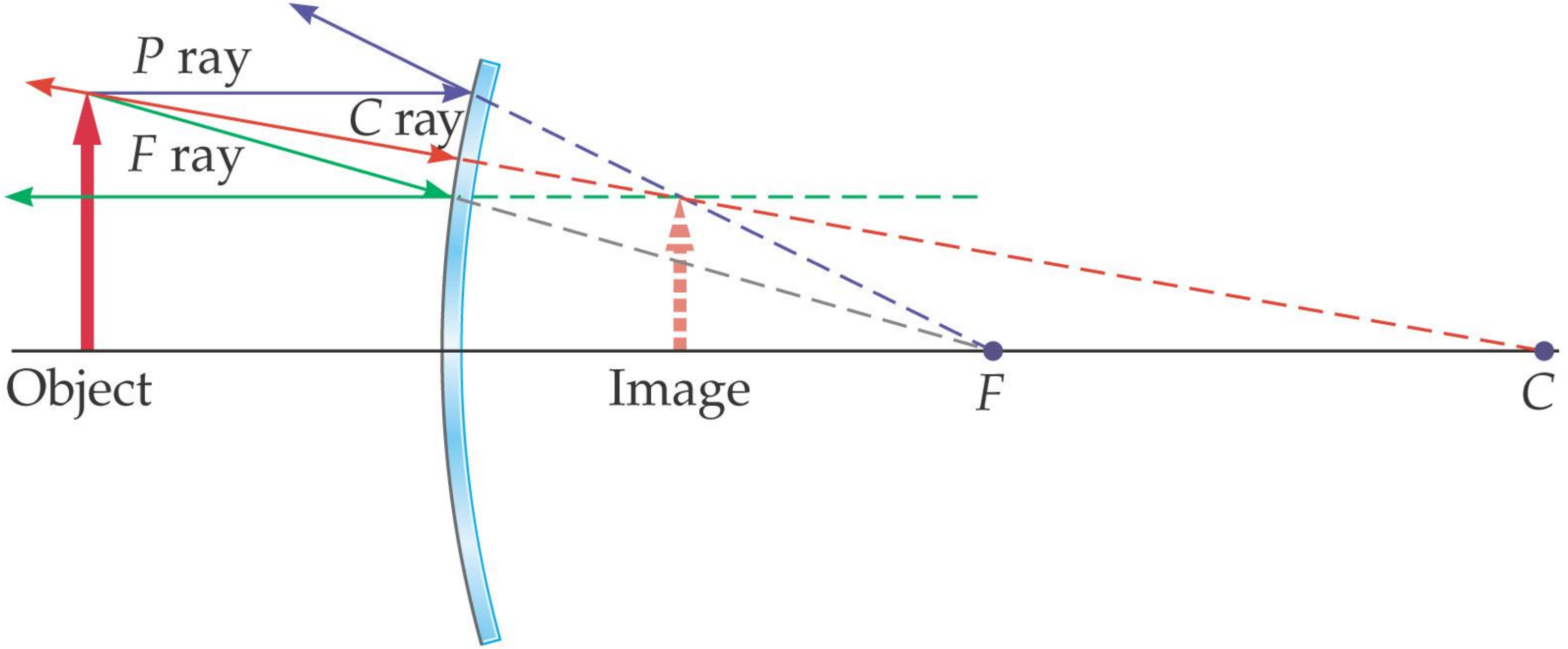
Ray Tracing and the Mirror Equation

These three rays are illustrated here.



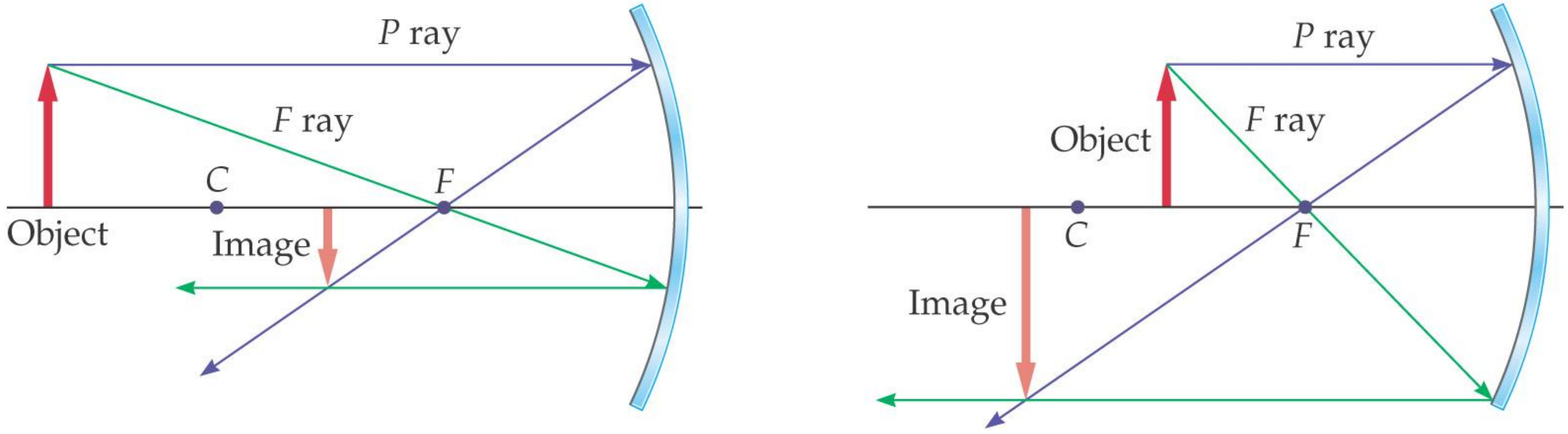
Ray Tracing and the Mirror Equation

This image shows how these three rays are used to find the image formed by a convex mirror. The image is located where the projections of the three rays cross. The size of the image can also be determined.



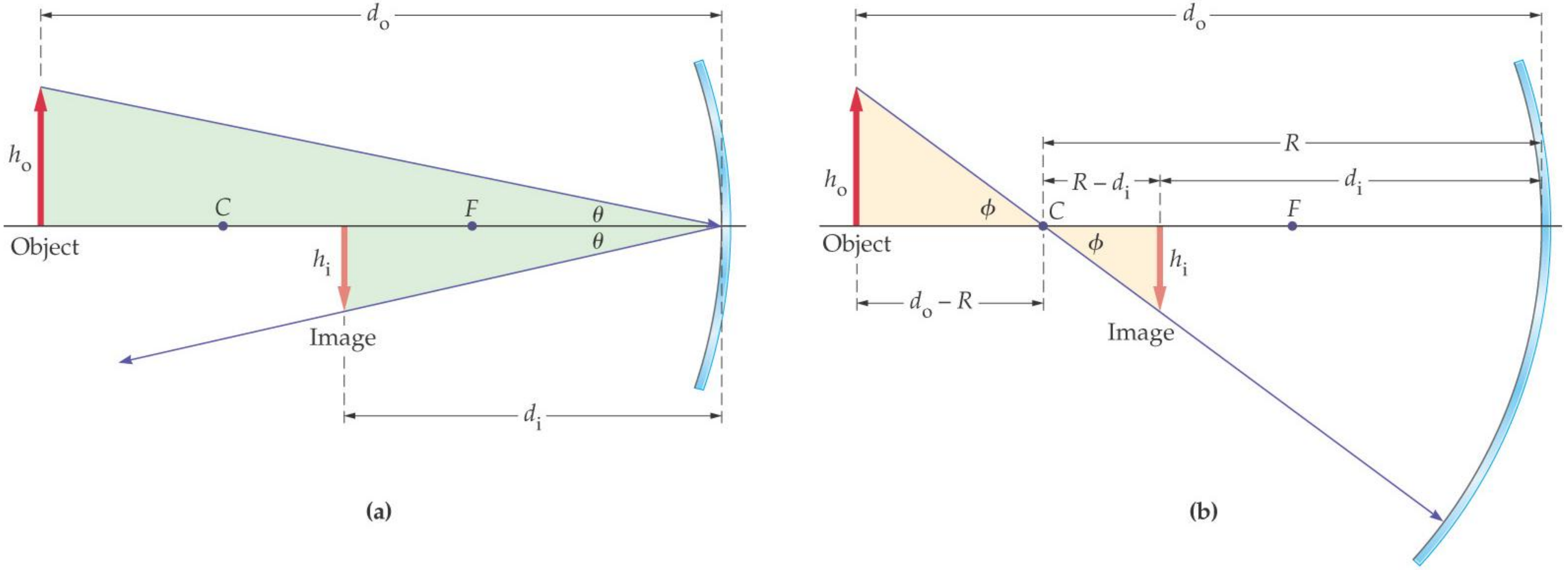
Ray Tracing and the Mirror Equation

The process is similar for a concave mirror, although there are different results depending on where the object is placed.



Ray Tracing and the Mirror Equation

We derive the mirror equation using the ray diagrams:



Ray Tracing and the Mirror Equation

Using the similar triangles and the fact that $f = \frac{1}{2} R$, we get the mirror equation:

The Mirror Equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Here, d_o is the distance from the mirror to the object, d_i is the distance from the mirror to the image, and f is the focal length.

Ray Tracing and the Mirror Equation

TABLE 26–1 Imaging Characteristics of Convex and Concave Spherical Mirrors

CONVEX MIRROR

Object location	Image orientation	Image size	Image type
Arbitrary	Upright	Reduced	Virtual

CONCAVE MIRROR

Object location	Image orientation	Image size	Image type
Beyond *C*	Inverted	Reduced	Real
C	Inverted	Same as object	Real
Between *F* and *C*	Inverted	Enlarged	Real
Just beyond *F*	Inverted	Approaching infinity	Real
Just inside *F*	Upright	Approaching infinity	Virtual
Between mirror and *F*	Upright	Enlarged	Virtual

Ray Tracing and the Mirror Equation

We can also find the magnification:

Magnification, *m*

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Ray Tracing and the Mirror Equation

Here are the sign conventions for concave and convex mirrors:

Focal Length

f is positive for concave mirrors.

f is negative for convex mirrors.

Magnification

m is positive for upright images.

m is negative for inverted images.

Image Distance

d_i is positive for images in front of a mirror (real images).

d_i is negative for images behind a mirror (virtual images).

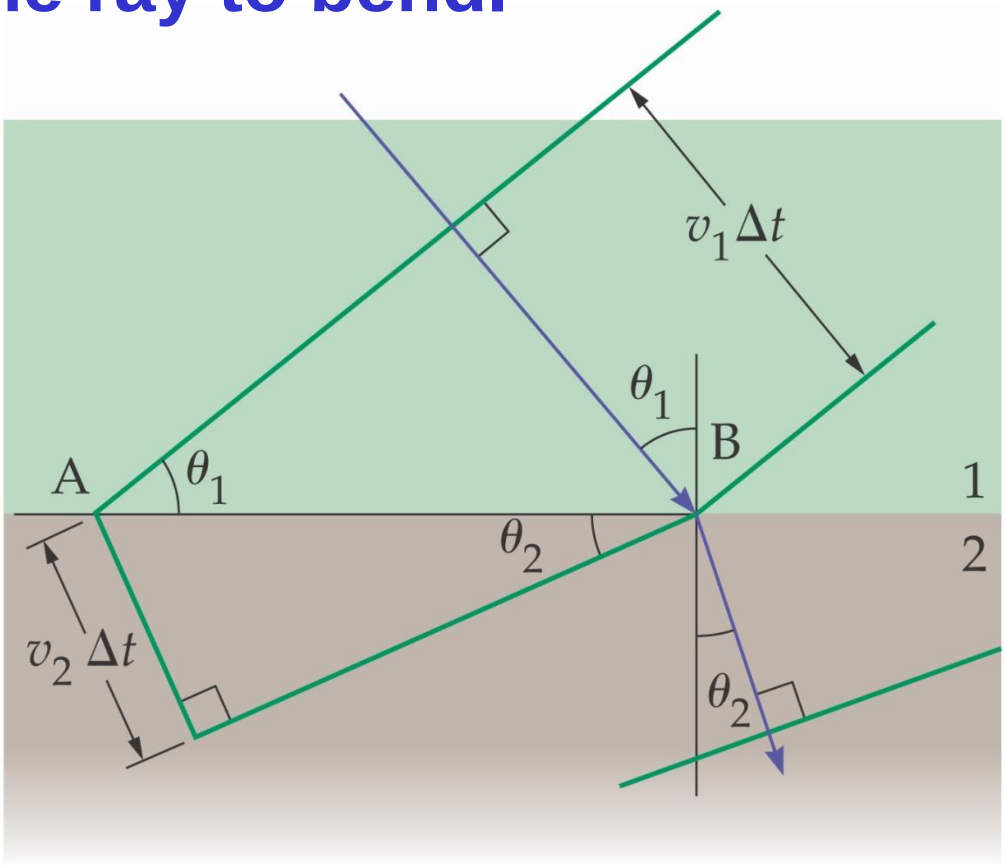
Object Distance

d_o is positive for objects in front of a mirror (real objects).

d_o is negative for objects behind a mirror (virtual objects).

The Refraction of Light

Light moves at different speeds through different media. When it travels from one medium into another, the change in speed causes the ray to bend.



The Refraction of Light

The angle of refraction is related to the different speeds:

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

The speed of light in a medium is given by the index of refraction of that medium:

Definition of the Index of Refraction, *n*

$$v = \frac{c}{n}$$

The Refraction of Light

Here are some typical indices of refraction:

TABLE 26–2 Index of Refraction for Common Substances

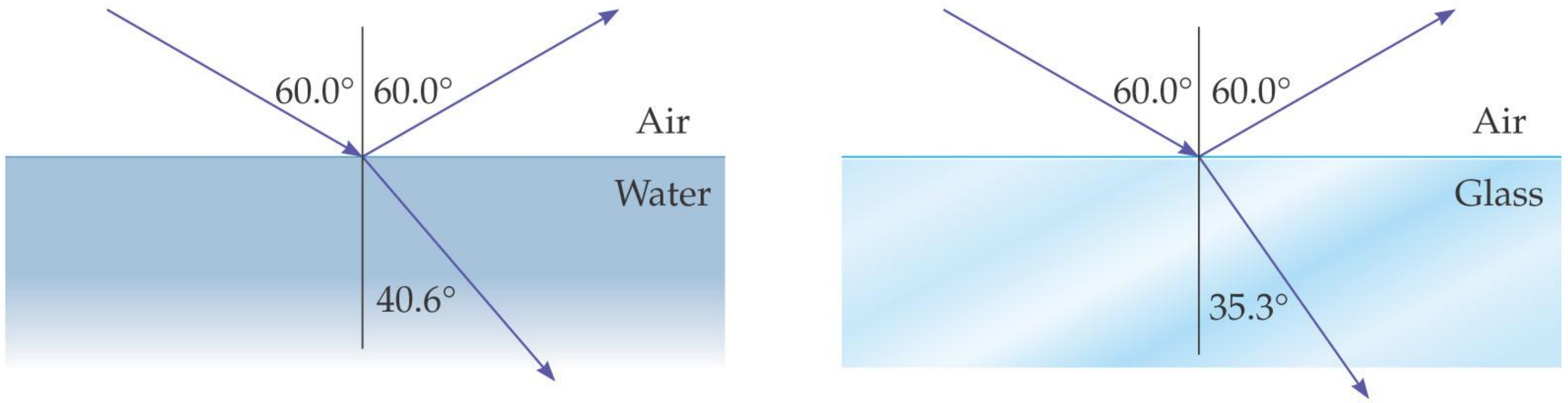
Substance	Index of refraction, *n*
Solids	
Diamond	2.42
Flint glass	1.66
Crown glass	1.52
Fused quartz (glass)	1.46
Ice	1.31
Liquids	
Benzene	1.50
Ethyl alcohol	1.36
Water	1.33
Gases	
Carbon dioxide	1.00045
Air	1.000293

The Refraction of Light

We can now write the angle of refraction in terms of the index of refraction:

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



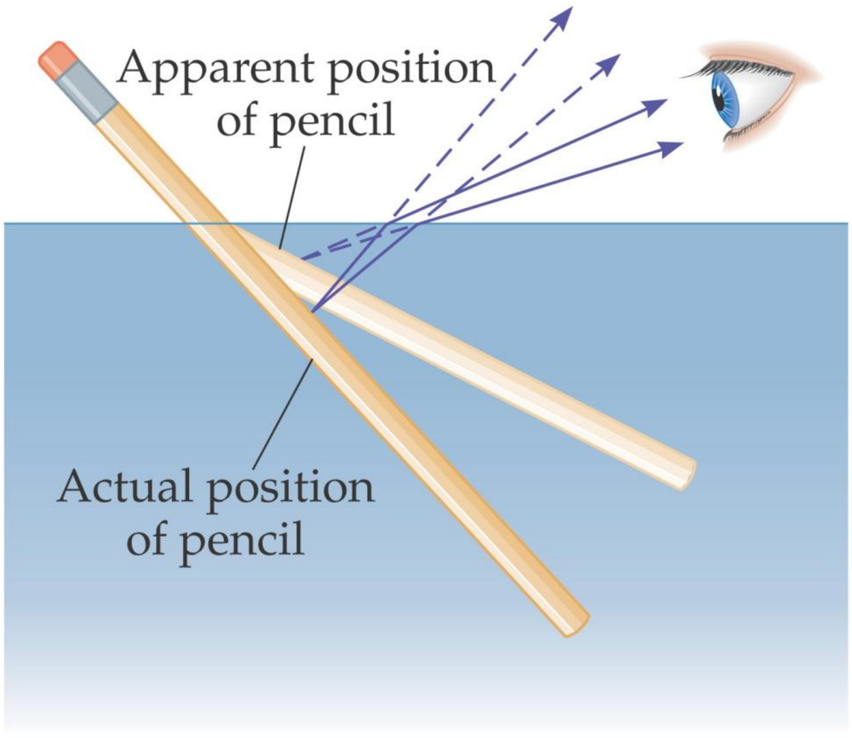
The Refraction of Light

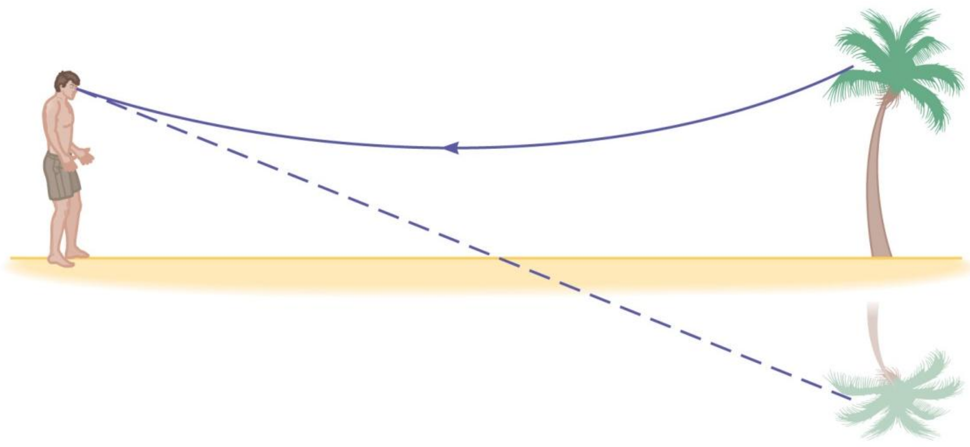
Basic properties of refraction:

- When a ray of light enters a medium where its speed *decreases*, it is bent *toward* the normal.
- When a ray of light enters a medium where its speed *increases*, it is bent *away* from the normal.
- There is no change in direction of propagation if there is no change in index of refraction. The greater the change in index of refraction, the greater the change in propagation direction.
- If a ray of light goes from one medium to another along the normal, it is undeflected, regardless of the index of refraction.

The Refraction of Light

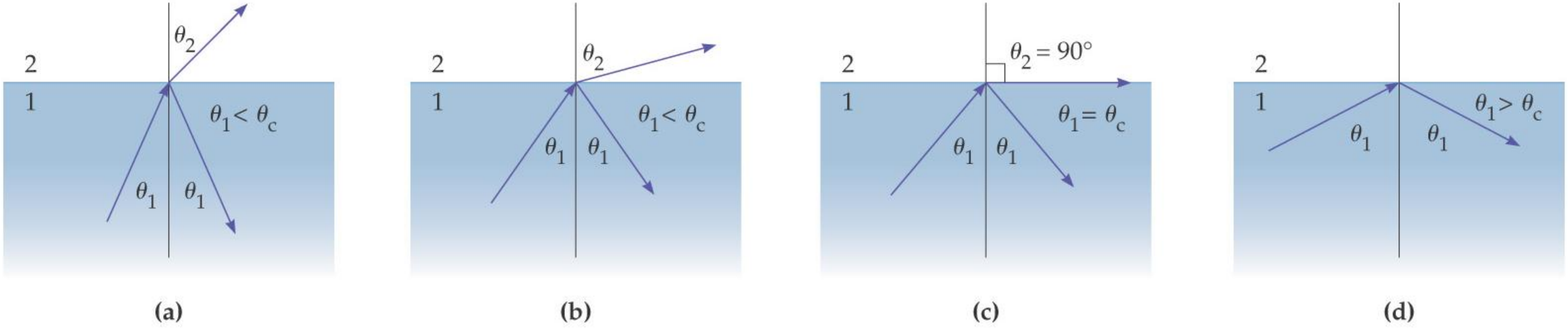
Refraction can make objects immersed in water appear broken, and can create mirages.





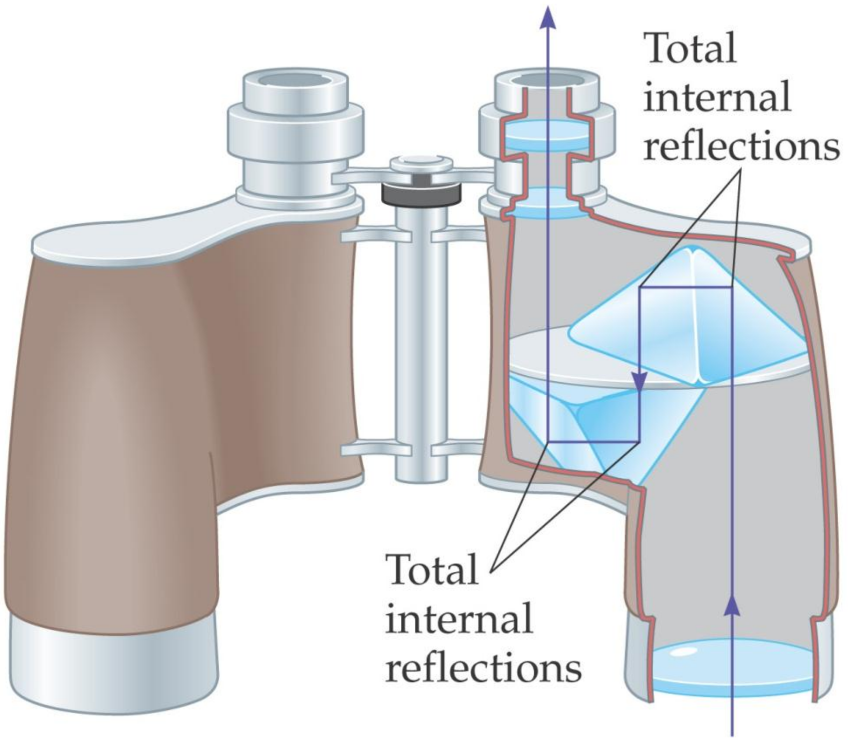
The Refraction of Light

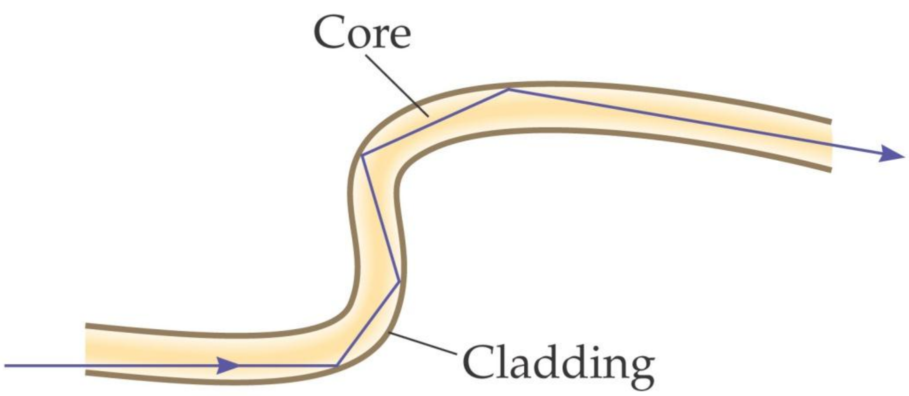
If light enters a medium of lower index of refraction, it will be bent away from the normal. If the angle of incidence is large enough, the angle of refraction is 90°; at larger incident angles the light will be totally reflected.



The Refraction of Light

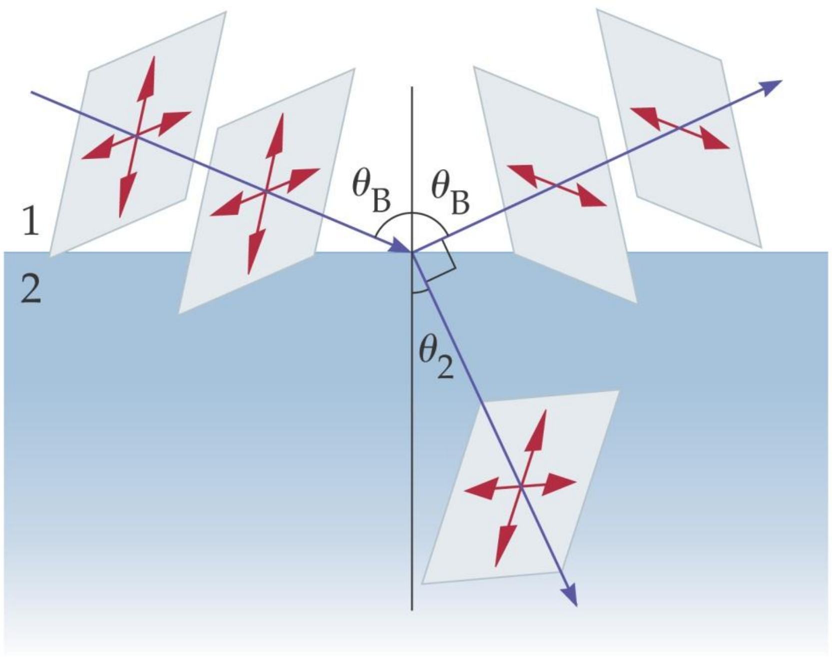
This is called total internal reflection, and the incident angle at which the angle of refraction is 90° is called the critical angle, θ_C. Total internal reflection is used in some binoculars and in optical fibers.





The Refraction of Light

There is a special angle called Brewster's angle; light reflected at this angle is totally polarized.



Reflected light is completely polarized when the reflected and refracted beams are at right angles to one another. The direction of polarization is parallel to the reflecting surface.

The Refraction of Light

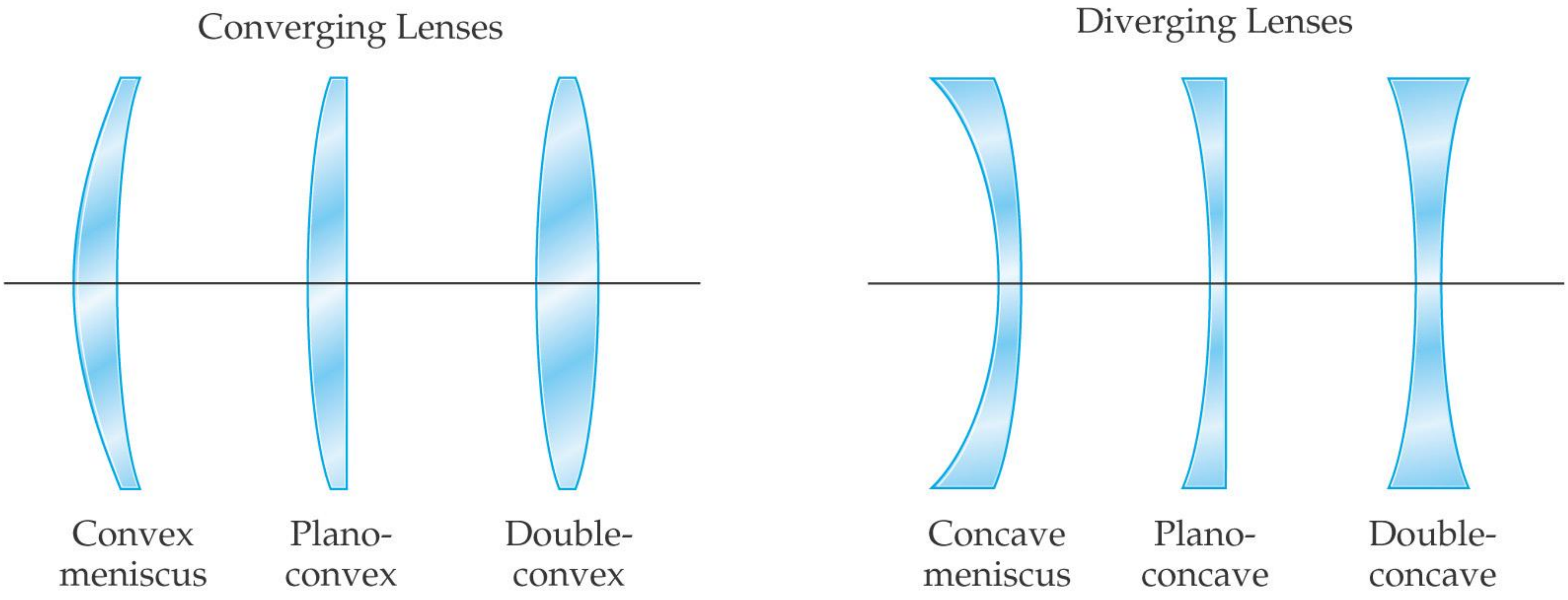
Brewster's angle can be calculated using the appropriate geometry:

Brewster's Angle, θ_B

$$\tan \theta_B = \frac{n_2}{n_1}$$

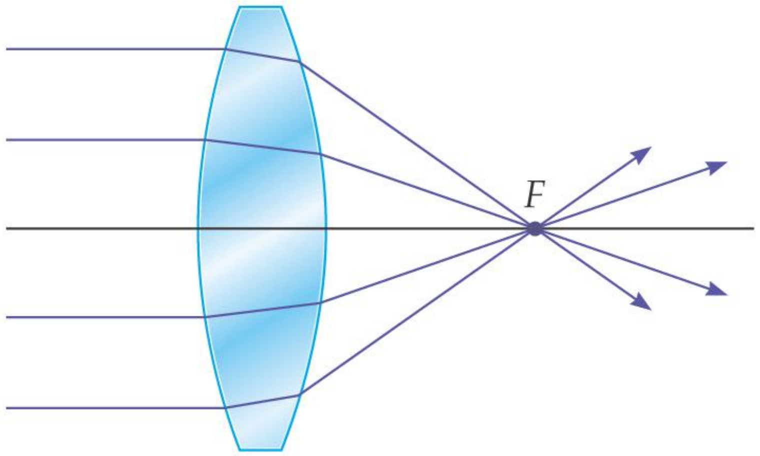
Ray Tracing for Lenses

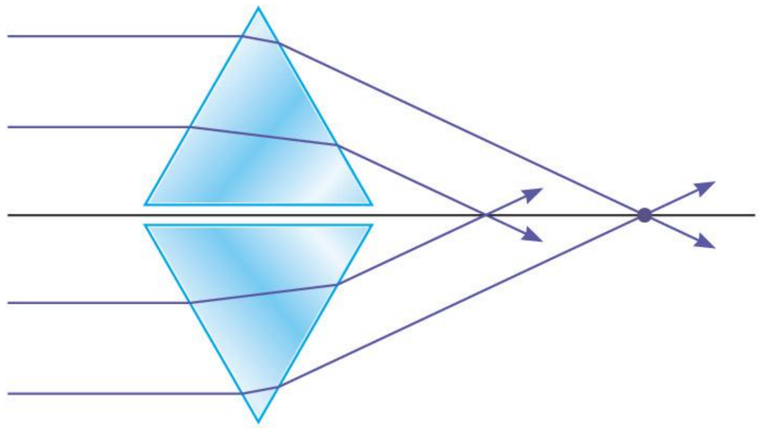
Lenses are used to focus light and form images. There are a variety of possible types; we will consider only the symmetric ones, the double concave and the double convex.



Ray Tracing for Lenses

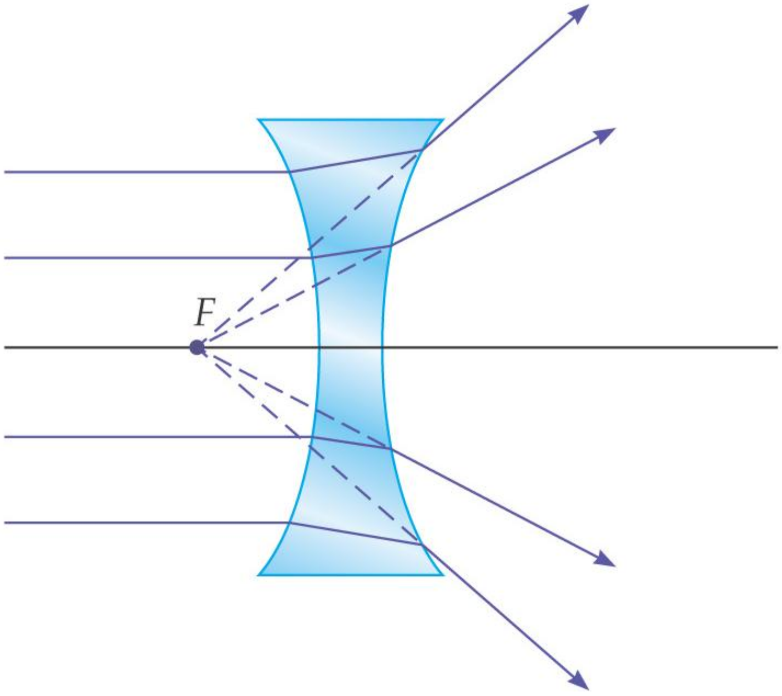
If we think of a convex lens as consisting of prisms, we can see how light going through it converges at a focal point (assuming the lens is properly shaped).

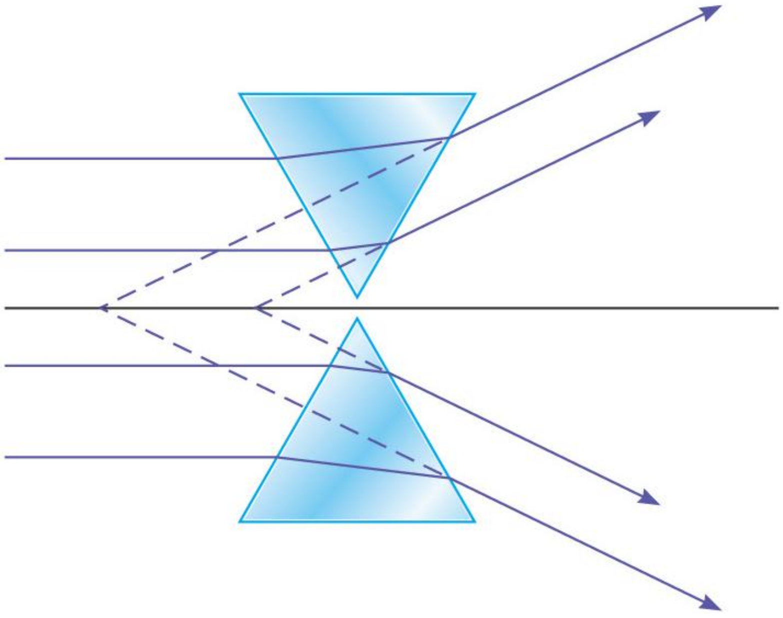




Ray Tracing for Lenses

A concave lens can also be modeled by prisms:





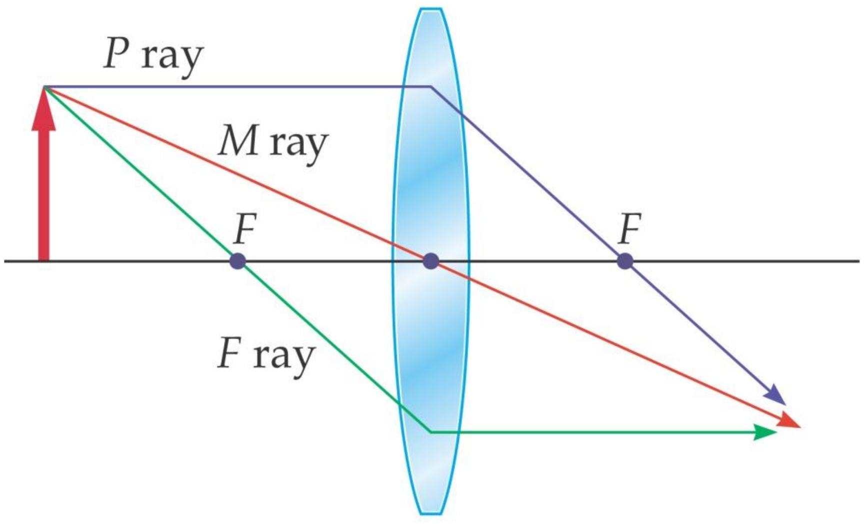
Ray Tracing for Lenses

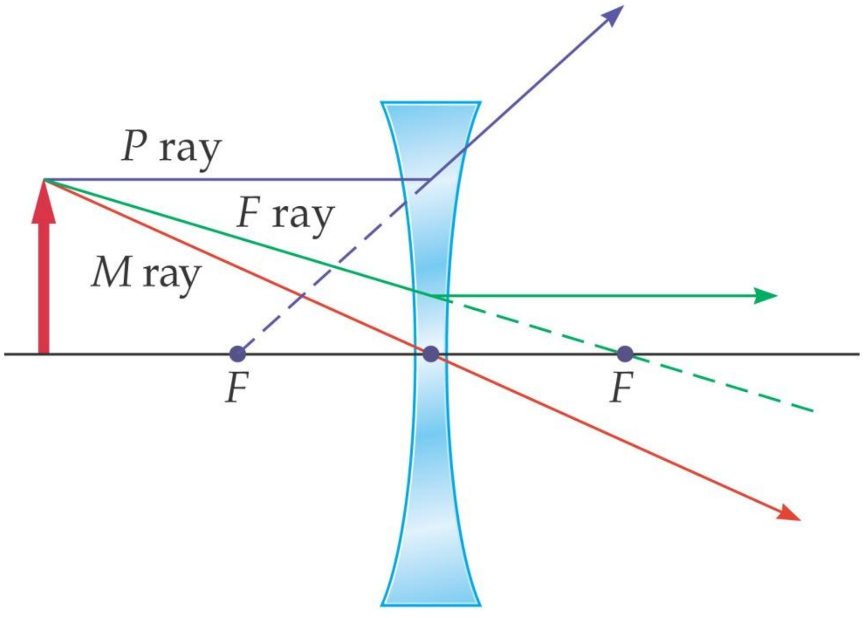
The three principal rays for lenses are similar to those for mirrors:

- **The *P* ray—or parallel ray—approaches the lens parallel to its axis.**
- **The *F* ray is drawn toward (concave) or through (convex) the focal point.**
- **The midpoint ray (*M* ray) goes through the middle of the lens. Assuming the lens is thin enough, it will not be deflected. This is the thin-lens approximation.**

Ray Tracing for Lenses

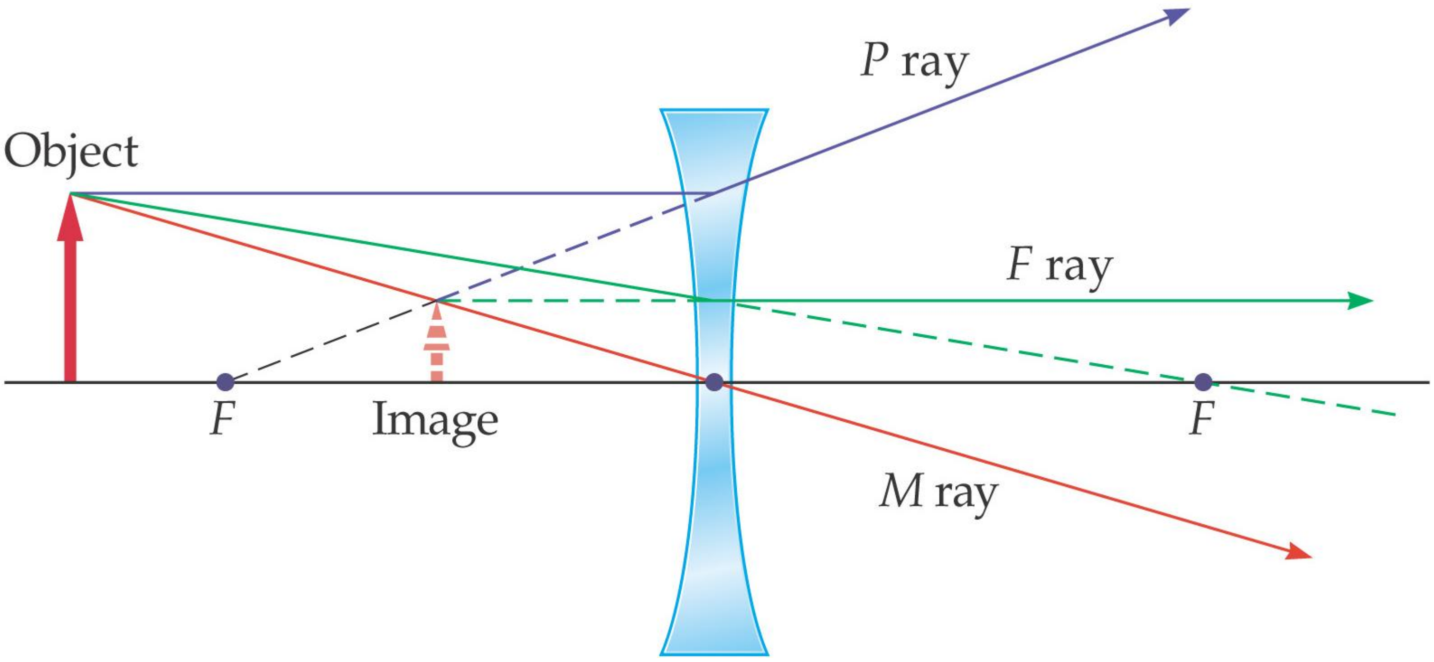
These diagrams show the principal rays for both types of lenses:





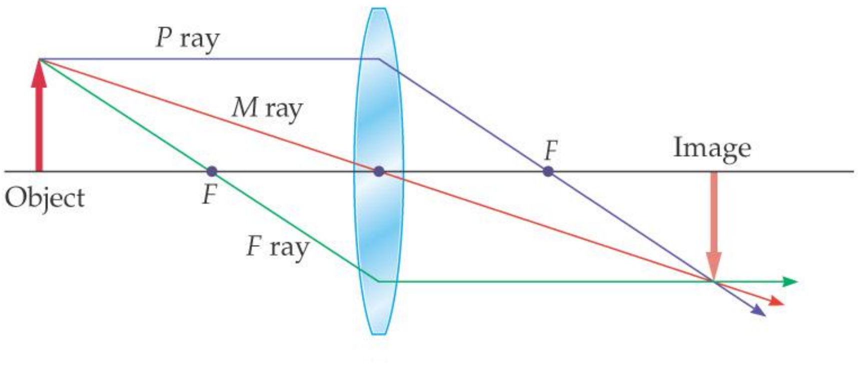
Ray Tracing for Lenses

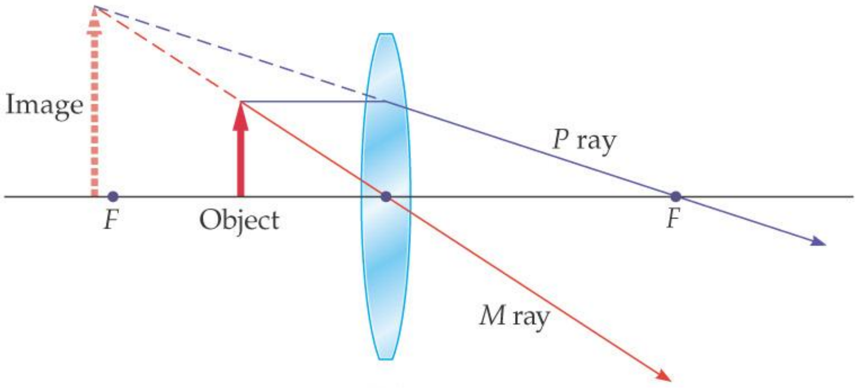
As with mirrors, we use these principal rays to locate the image:



Ray Tracing for Lenses

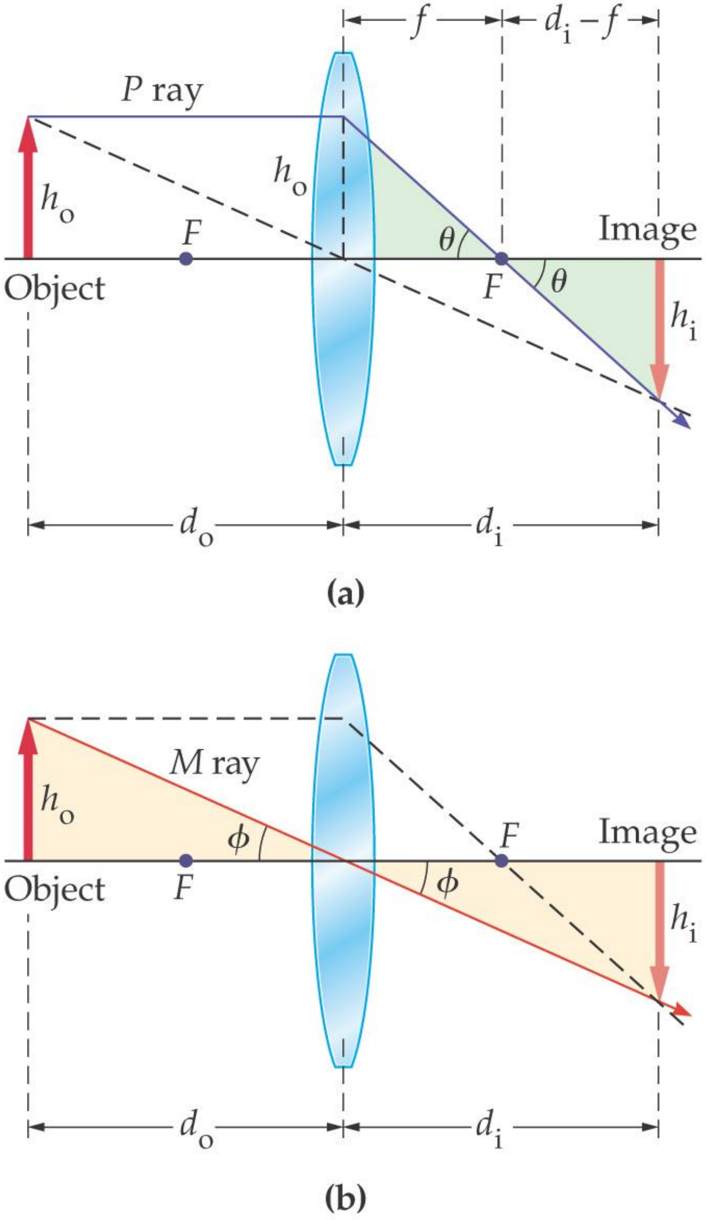
The convex lens forms different image types depending on where the object is located with respect to the focal point:





The Thin-Lens Equation

We derive the thin-lens equation in the same way we did the mirror equation, using these diagrams:



The Thin-Lens Equation

This gives us the thin-lens approximation, as well as the magnification:

Thin-Lens Equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Magnification, *m*

$$m = -\frac{d_i}{d_o}$$

The Thin-Lens Equation

Sign conventions for thin lenses:

Focal Length

f is positive for converging (convex) lenses.

f is negative for diverging (concave) lenses.

Magnification

m is positive for upright images (same orientation as object).

m is negative for inverted images (opposite orientation of object).

Image Distance

d_i is positive for real images (images on the opposite side of the lens from the object).

d_i is negative for virtual images (images on the same side of the lens as the object).

Object Distance

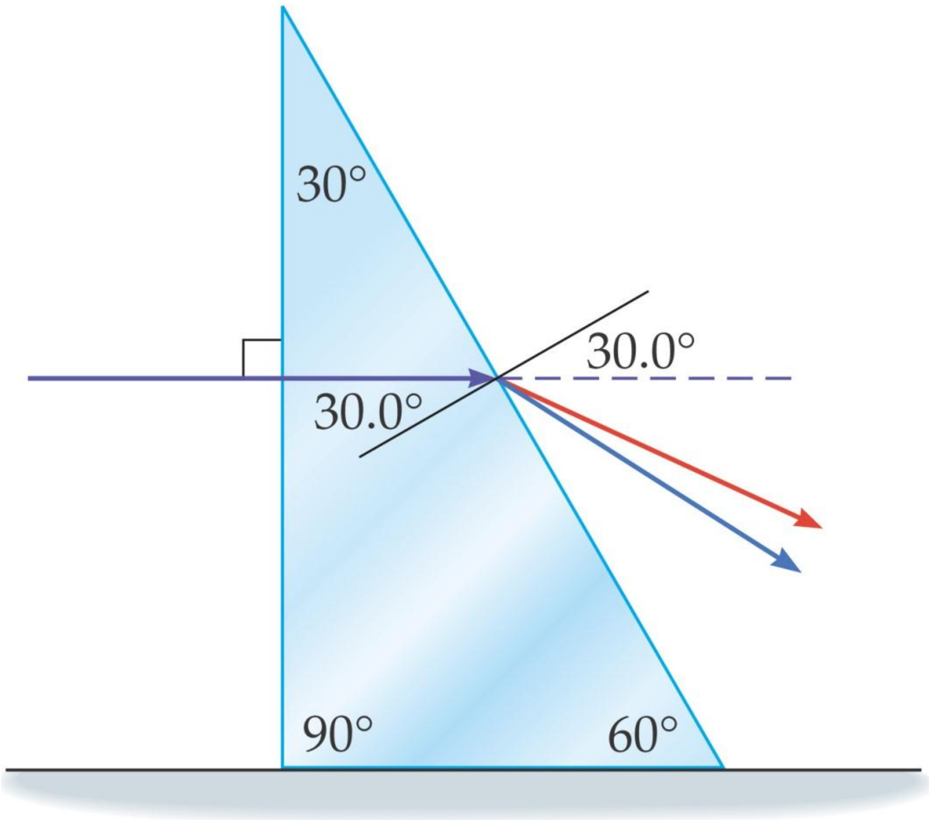
d_o is positive for real objects (from which light diverges).

d_o is negative for virtual objects (toward which light converges).

Dispersion and the Rainbow

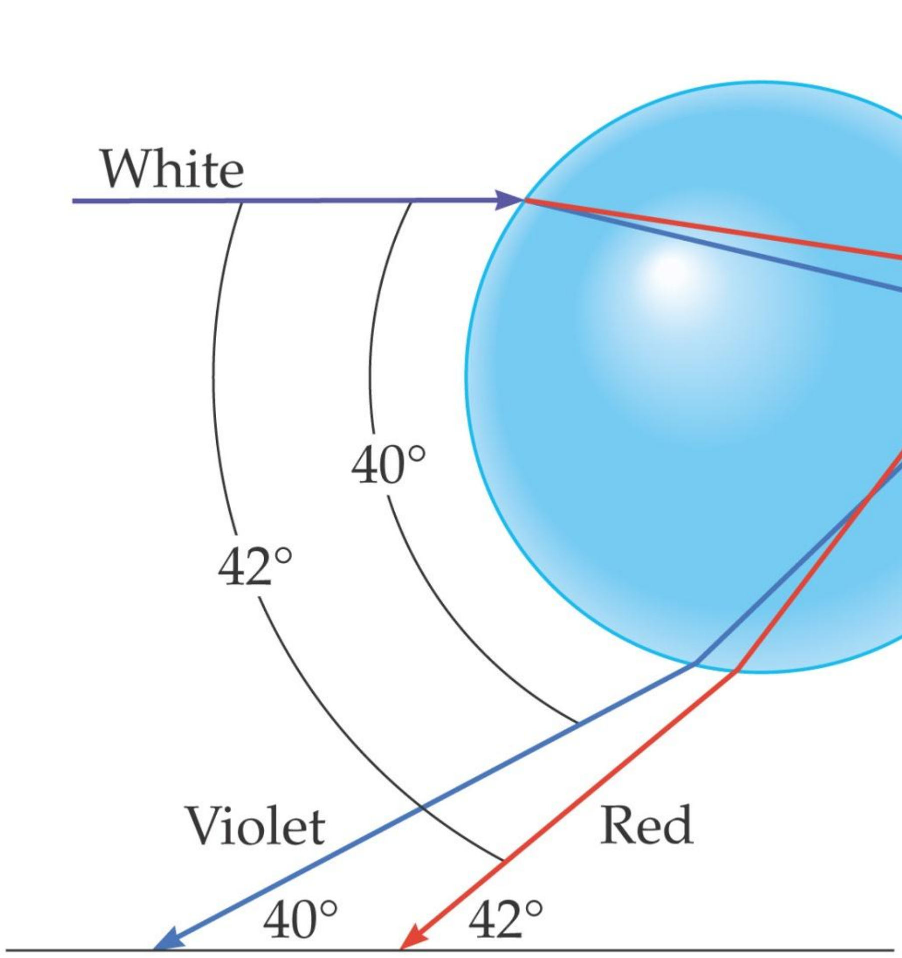
The index of refraction varies slightly with the frequency of light; in general, the higher the frequency, the higher the index of refraction.

This means that refracted light is "spread out" in a rainbow of colors; this phenomenon is known as dispersion.



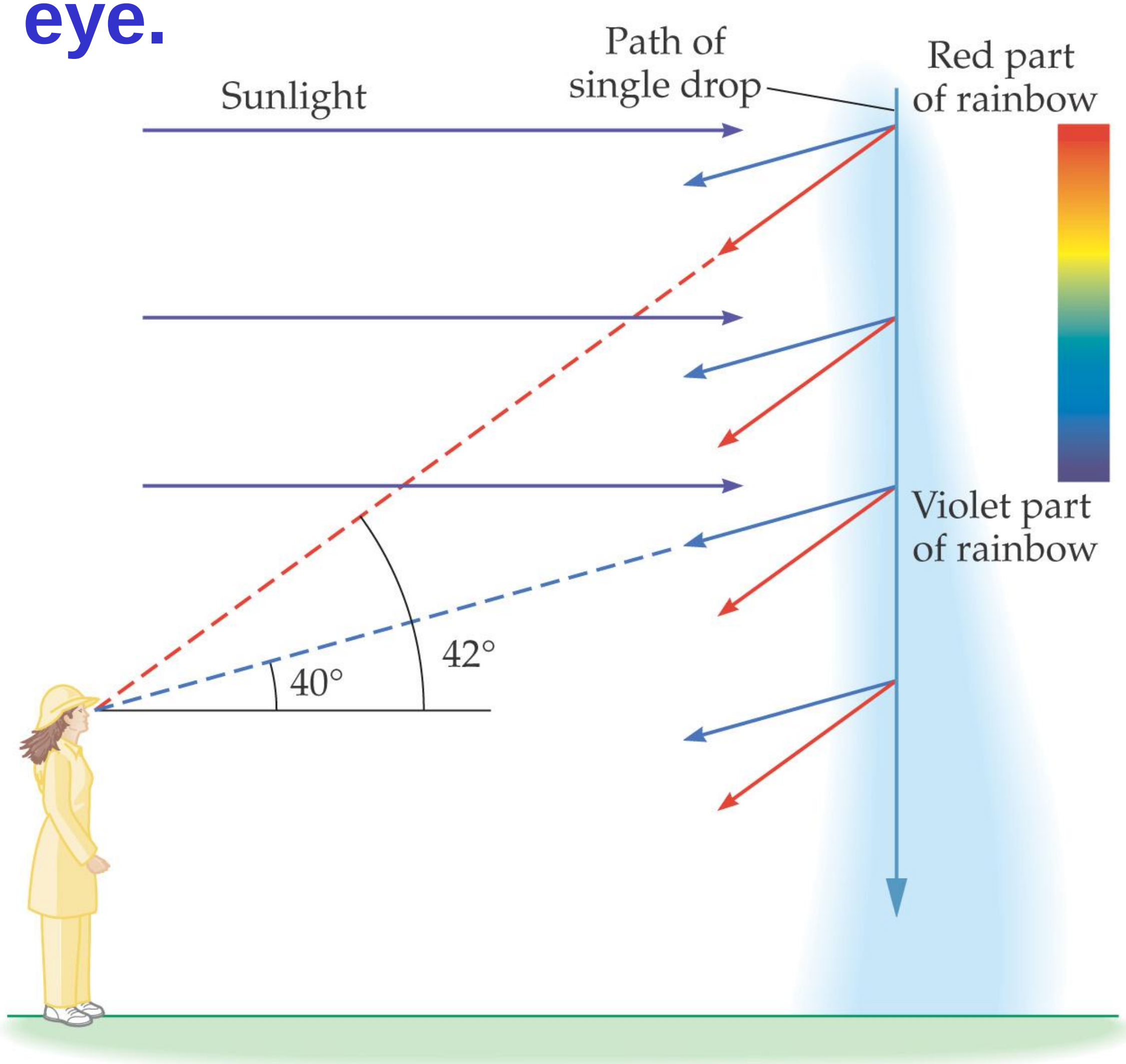
Dispersion and the Rainbow

Rainbows are created by the dispersion of light as it refracts in a rain drop.



Dispersion and the Rainbow

As the drop falls, all the colors of the rainbow arrive at the eye.



Dispersion and the Rainbow

Sometimes a faint secondary arc can be seen.

Summary

- A wave front is a surface along which the wave phase is constant. Rays, perpendicular to the wave fronts, indicate the direction of propagation.
- The angle of incidence equals the angle of reflection.
- The image formed by a plane mirror is upright, but appears reversed left to right; appears to be the same distance behind the mirror as the object is in front of it; and is the same size as the object.

Summary Contd..

- Spherical mirrors have spherical reflecting surfaces. A concave mirror is curved inward, and a convex one outward.
- Focal length of a convex mirror: $f = -\frac{1}{2}R$
- Focal length of a concave mirror: $f = \frac{1}{2}R$
- An image is real if light passes through it, virtual if it does not.
- Mirror equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

Summary Contd…

- **Magnification:** $$m = -\frac{d_i}{d_o}$$

- **Refraction is the change in direction of light due to a change in speed.**

- **The index of refraction gives the speed of light in a medium:**

$$v = \frac{c}{n}$$

Summary Contd…

- **Snell's law:** $n_1 \sin\theta_1 = n_2 \sin\theta_2$

- **Light entering a medium of higher *n* is bent towards the normal; light entering a medium of lower *n* is bent away from the normal.**

- **When light enters a medium of lower *n*, there is a critical angle beyond which the light will be totally reflected.**

$$\sin\theta_c = \frac{n_2}{n_1}$$

Summary Contd….

- At Brewster's angle, the reflected light is totally polarized:

$$\tan \theta_B = \frac{n_2}{n_1}$$

- A lens uses refraction to bend light and form images.
- Thin-lens equation: $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Summary Contd….

- **Magnification:** $$m = -\frac{d_i}{d_o}$$

- **The index of refraction varies with frequency; different frequencies of light are bent different amounts. This is called dispersion.**

Thank You